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Finite groups in which two different Sylow p-subgroups have trivial intersection for an odd prime p

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1. Introduction

Let p be a prime and let G be a finite group which satisfies the following condition.

(TIp): two different Sylow p-groups contain only the identity element in common.

Suzuki [**10**] treated the case $p=2$. When p is an odd prime, it seems quite difficult to describe all possibilities. Here we treat a restricted case that G posess a p-non-stable faithful representation. More precisely, we say that G is a (Qp)-group if G satisfies the following condition:

(Qp): There exists a finite vector space M over $GF(p)$, the field with p elements, such that M is a faithful $GF(p)(G)$-module and some nontrivial element of G has minimal polynomial $(X-1)^2$ over M.

We remark that the above condition (Qp) is always valid for $p=2$ when G is of even order. This can be seen by taking M to be the group algebra of G over $GF(2)$ and let G act on M naturally. The main result of this paper is the following theorem.

THEOREM 1. *Let p be an odd prime and let G be a finite group satisfy the conditions (TIp) and (Qp). Then one of the following holds:*

(a) *A Sylow p-group of G is a normal subgroup.*

(b) *G contains normal subgroups G_1 and G_2 such that*

$$G \geqq G_1 > G_2 \geqq 1$$

where G_2 is the center of G_1, both G/G_1 and G_2 are of order prime to p and G_1/G_2 is isomorphic to $L_2(p^n)$ or $U_3(p^n)$ for some positive integer n.

(c) *$p=3$ and G contains normal subgroups G_1 and G_2 such that*

$$G \geqq G_1 > G_2 \geqq 1$$

where G_2 is the maximal normal 2-group of G_1, G/G_1 has order prime to p and G_1/G_2 is isomorphic to the cyclic group of order 3 or A_5.

Let K/Ω be an algebraic function field with one variable of genus $g>1$