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Evolution equations associated with the subdifferentials

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§ 0. Introduction.

In this paper we consider the nonlinear evolution equation of the form

$$(E) \qquad du(t)/dt+\partial\phi^{t}(u(t))\ni f(t) \qquad 0\leqq t\leqq T\,,$$

in a real Hilbert space H. Here, for almost every $t\in[0, T]$, $\partial\phi^{t}$ is the subdifferential of a lower semicontinuous convex function ϕ^{t} from H into $]-\infty, \infty]$ ($\phi^{t}\not\equiv+\infty$).

Since Brézis [2] first treated the equation (E) in the case $\phi^{t}=\phi$ is independent of t, many authors have investigated the existence, uniqueness and regularity of solutions of (E). (See Attouch and Damlamian [1], Kenmochi [5], Maruo [6], Watanabe [8], Yamada [10], [11], etc.)

This paper establishes an existence, uniqueness theorem for strong solutions of (E) under relatively weak assumptions on the t-dependence of ϕ^{t} generalizing the results of [1], [5], [6], [8], [10] and [11]. We employ the method of Kenmochi [5], that is, we would like to approximate (E) by difference approximations with respect to the time. We also use the idea of Maruo [6] under these hypotheses to establish estimates for solutions of the approximation schemes. The main advance over [10, 11] is the relaxation of a hypothesis on the t-dependence of the ϕ^{t} from absolute continuity to bounded variation.

The contents of this paper are as follows. § 1 recalls the basic properties of a lower semicontinuous convex function ϕ. In § 2 we list the basic hypotheses and state the existence theorem for (E). § 3–7 comprise the proof of the theorem. § 3 shows the measurability of $\phi^{\cdot}(v(\cdot))$ for any strongly measurable function v. In § 4 we prepare some lemmas which play important roles in § 5. In § 5 we drive recursive inequalities for solutions of the approximation schemes and establish estimates for them. In § 6 we prove that the approximate solutions converge as the mesh of the partitions approaches zero. Then we get the local existence of the strong solution. In § 7 we prove the global existence of it.

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