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# Homogeneous Riemannian manifolds with a fixed isotropy representation

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## 1. Introduction.

In this paper we give a classification of simply-connected homogeneous Riemannian manifolds $M=G/H$ where $H$ is isomorphic to a product of rotation groups and the linear isotropy representation of $H$ is a direct sum of standard representations with a trivial representation. This situation arises naturally in the study of homogeneous Riemannian manifolds which admit a large group of isometries. In fact if $M=I_0(M)/H$, where

$$\dim I(M)>\frac{n^2}{4}+n\,,\quad n=\dim M\geqq 11 \tag{1.1}$$

then it follows that $H\cong SO(k)\times K$, with $k>n/2$, $K\subseteqq SO(n-k)$ and the linear isotropy representation of $H$ splits [**5**, Theorem 1.18].

Our results are quite simple to state if each of the rotation groups has order at least 3. In that case $M$ is isometric to a product of a certain number of simply-connected manifolds of constant curvature together with a simply-connected Lie group with a left-invariant metric. (Theorem B). If $H$ is isomorphic to a single rotation group, this appears to be consistent with some local results obtained by Kurita [8] a number of years ago. If some of the rotation groups in the decomposition of $H$ have order 2, then the description of the corresponding manifolds becomes more complicated. This is done in Section 4, where, in particular, we obtain a generalization of Cartan's classification [3] of 3-dimensional manifolds which admit a transitive group of motions of dimension 4.

In Section 5, we apply the above results to give an explicit description of those manifolds satisfying (1.1) and $n-3\leqq k\leqq n$. This turns up some inaccuracies and extends some results in [7], while at the same time exhibiting the differences with the compact case studied by Lukesh. In [**9**] it is shown that if $M$ is compact and satisfies (1.1), then it must split isometrically with one factor being a standard sphere $S^k$, $k>n/2$. As we shall see in Section 5, there

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