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Rate of decay of local energy and wave operators for symmetric systems

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§1. Introduction.

A large class of wave propagation phenomena of classical physics and quantum mechanics are governed by "symmetric systems" of partial differential equations of the form

$$\frac{1}{i}\frac{\partial u}{\partial t}=M(x)(P(D)+\sum_{j=1}^{K} q_j(x)Q_j(D))u\,. \tag{1.1}$$

Here $x\in \boldsymbol{R}^n$, $t\in \boldsymbol{R}$, $D=-i\partial/\partial x$, $u(x, t)$ is a $\boldsymbol{C}^m$-valued function, $P(D)+\sum_{j=1}^{K}q_j(x)Q_j(D)$ is a self-adjoint differential operator in $[L_2(\boldsymbol{R}^n)]^m$, and $M(x)$ is an $m\times m$ Hermitian matrix with

$$C|\xi|^2\leqq(M(x)\xi, \xi)\leqq C^{-1}|\xi|^2, \qquad x, \xi\in \boldsymbol{R}^n \tag{1.2}$$

for some positive constant C.

In this paper we study the asymptotic behavior as $t\rightarrow\infty$ of the solution of the system (1.1) with initial value having finite energy. In doing so, we compare the system (1.1) with the unperturbed system

$$\frac{1}{i}\frac{\partial u}{\partial t}=P(D)u\,, \tag{1.3}$$

assuming that for some $s>1$ and $C>0$

$$|M(x)-I|+\sum_{j=1}^{K}|q_j(x)|\leqq C(1+|x|^2)^{-s/2}, \qquad x\in \boldsymbol{R}^n\,. \tag{1.4}$$

Here I is the unit matrix, and $|A|$ denotes the norm of an $m\times m$ matrix A: $|A|=(\sum_{i,j=1}^{m}|A_{ij}|^2)^{1/2}$.

Let $\boldsymbol{H}_0$ and $\boldsymbol{H}$ be Hilbert spaces with inner products

$$(f, g)_{\boldsymbol{H}_0}=\int_{\boldsymbol{R}^n} f(x)\overline{g(x)}dx, \qquad f, g\in[L_2(\boldsymbol{R}^n)]^m \tag{1.5}$$

and

$$(f, g)_{\boldsymbol{H}}=\int_{\boldsymbol{R}^n} M(x)^{-1}f(x)\overline{g(x)}dx\,, \qquad f, g\in[L_2(\boldsymbol{R}^n)]^m\,, \tag{1.6}$$