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On the bifurcation of the multiplicity and topology of the Newton boundary

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0. Introduction.

In [3], A. G. Kouchnirenko presented a beautiful formula about the multiplicity of an isolated singularity of a hypersurfaces $V=f^{-1}(0)$ in $\boldsymbol{C}^n$ where $f(z)$ is assumed to have a non-degenerate Newton principal part. It states that the multiplicity of V at the origin (=the Milnor number) is equal to the Newton number of $\Gamma_-(f)$ (and thus is independent of a particular choice of the coefficients of $f(z)$).

The purpose of this paper is to give a geometrical proof of Kouchnirenko's Theorem from the viewpoint of the bifurcation of the multiplicity. First we prove that *the Milnor fibration of $f(z)$ is determined by the Newton boundary $\Gamma(f)$ if the Newton principal part of f is non-degenerate* (Theorem 2.1).

For the calculation of the multiplicity, we consider the bifurcating equation:

$$z_1\frac{\partial f}{\partial z_1}-t\gamma_1=\cdots=z_n\frac{\partial f}{\partial z_n}-t\gamma_n=0\,.$$

If $\gamma=(\gamma_1, \cdots, \gamma_n)$ is generic and t is sufficiently small, the bifurcating solutions of the above equation are all simple and one finds exactly $n!$ volume $\Gamma_-(f)$ *solutions* ($t\neq 0$) (Theorem 4.2).

1. Milnor fibration.

Let $f(z_1, z_2, \cdots, z_n)$ be an analytic function in an open neighbourhood U of $\boldsymbol{C}^n$ ($f(0)=0$) and assume that $f(z)$ has an isolated critical point at the origin. We can take a positive number ε so that the sphere $S(r)=\{z\in\boldsymbol{C}^n\,;\ \|z\|^2=|z_1|^2+\cdots+|z_n|^2=r^2\}$ cuts the hypersurface $V_0=f^{-1}(0)$ transversely for any $0<r\leqq\varepsilon$. (Therefore $V_0\cap S(r)$ is a smooth manifold.) Fixing such an ε, we can take $\delta>0$ such that $V_\eta=f^{-1}(\eta)$ is non-singular in $D(\varepsilon)$ and is transverse to $S(\varepsilon)$ for $0<|\eta|\leqq\delta$ where $D(\varepsilon)=\{z\in\boldsymbol{C}^n\,;\ \|z\|\leqq\varepsilon\}$. Then we have a so-called Milnor

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