Weil’s representations of the symplectic groups over finite fields*

By Hiroyuki YOSHIDA

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Introduction.

Let $F(q)$ be the finite field with $q$ elements where $q$ is odd. Suppose that there is given a $2n \times 2n$ symmetric matrix $S$ whose entries are in $F(q)$ such that $\det S \neq 0$. Let $O_q(S)$ denote the special orthogonal group with respect to $S$ and $Sp(2m)$ denote the symplectic group of genus $m$. We consider $O_q(S)$ and $Sp(2m)$ as connected semisimple algebraic groups defined over $F(q)$ endowed with the Frobenius map $F$. Let $M_{2n,m}(F(q))$ be the set of all $2n \times m$ matrices with entries in $F(q)$ and $S(M_{2n,m}(F(q)))$ be the space of all complex valued functions on $M_{2n,m}(F(q))$. Then we can construct, associated with $S$, so called Weil’s representation $\pi_{S,m}$ of $Sp(2m)^F$ realized on $S(M_{2n,m}(F(q)))$. The representation $\pi_{S,m}$ can be decomposed naturally according to representations of $O_q(S)^F$. Thus we have a correspondence from the set of the equivalence classes of all representations of $O_q(S)^F$ to that of $Sp(2m)^F$. For a representation $\rho$ of $O_q(S)^F$, let $\pi_{S,m}(\rho)$ denote the representation of $Sp(2m)^F$ which corresponds to $\rho$.

The purpose of this paper is to get some insight about the nature of this correspondence in the case $m=n$. A natural parametrization of most of the irreducible representations of $O_q(S)^F$ and $Sp(2m)^F$ is available from the work of Deligne-Lusztig [4]. In their paper, for an arbitrary connected reductive algebraic group $G$ defined over $F(q)$, a maximal $F$-stable torus $T$ and a character $\theta$ of $T^F$, a virtual representation $R_\theta^T$ of $G^F$ is constructed. Moreover it is shown that any irreducible representation of $G^F$ occurs as a constituent of some $R_\theta^T$ and that $(-1)^{\sigma(G)-\sigma(T)}R_\theta^T$ is an irreducible representation if $\theta$ is in general position, where $\sigma(G)$ and $\sigma(T)$ denote the $F(q)$-rank of $G$ and $T$ respectively. Now let $T$ be a maximal $F$-stable torus of $O_q(S)$. Then there exists a maximal $F$-stable torus $T'$ of $Sp(2m)$ such that $T$ is isomorphic to $T'$ over $F(q)$ as algebraic tori. We fix the isomorphism between $T^F$ and $T'^F$, which is similar to that between $T^F_0$ and $T'^F_0$ given in §2. Let $\theta$ be a character of $T^F$ which

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