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Weil's representations of the symplectic groups over finite fields*)

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Introduction.

Let $\boldsymbol{F}(q)$ be the finite field with q elements where q is odd. Suppose that there is given a $2n\times 2n$ symmetric matrix S whose entries are in $\boldsymbol{F}(q)$ such that $\det S\neq 0$. Let $O_1(S)$ denote the special orthogonal group with respect to S and $Sp(2m)$ denote the symplectic group of genus m. We consider $O_1(S)$ and $Sp(2m)$ as connected semisimple algebraic groups defined over $\boldsymbol{F}(q)$ endowed with the Frobenius map F. Let $M_{2n,m}(\boldsymbol{F}(q))$ be the set of all $2n\times m$ matrices with entries in $\boldsymbol{F}(q)$ and $\mathcal{S}(M_{2n,m}(\boldsymbol{F}(q)))$ be the space of all complex valued functions on $M_{2n,m}(\boldsymbol{F}(q))$. Then we can construct, associated with S, so called Weil's representation $\pi_{S,m}$ of $Sp(2m)^F$ realized on $\mathcal{S}(M_{2n,m}(\boldsymbol{F}(q)))$. The representation $\pi_{S,m}$ can be decomposed naturally according to representations of $O_1(S)^F$. Thus we have a correspondence from the set of the equivalence classes of all representations of $O_1(S)^F$ to that of $Sp(2m)^F$. For a representation ρ of $O_1(S)^F$, let $\pi_{S,m}(\rho)$ denote the representation of $Sp(2m)^F$ which corresponds to ρ.

The purpose of this paper is to get some insight about the nature of this correspondence in the case $m=n$. A natural parametrization of most of the irreducible representations of $O_1(S)^F$ and $Sp(2n)^F$ is available from the work of Deligne-Lusztig [4]. In their paper, for an arbitrary connected reductive algebraic group G defined over $\boldsymbol{F}(q)$, a maximal F-stable torus T and a character θ of T^F, a virtual representation R_T^θ of G^F is constructed. Moreover it is shown that any irreducible representation of G^F occurs as a constituent of some R_T^θ and that $(-1)^{\sigma(G)-\sigma(T)}R_T^\theta$ is an irreducible representation if θ is in general position, where $\sigma(G)$ and $\sigma(T)$ denote the $\boldsymbol{F}(q)$-rank of G and T respectively. Now let T be a maximal F-stable torus of $O_1(S)$. Then there exists a maximal F-stable torus T' of $Sp(2n)$ such that T is isomorphic to T' over $\boldsymbol{F}(q)$ as algebraic tori. We fix the isomorphism between T^F and T'^F, which is similar to that between $T_0{}^F$ and $T_1{}^F$ given in § 2. Let θ be a character of T^F which

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