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The subsequentiality of product spaces

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§ 1. Introduction.

A space is said to be a *subsequential space* if it can be embedded as a subspace of a sequential space. The closed image of a metric space is shortly said to be a *Lašnev space* (cf. [4], [5]).

Professor K. Nagami posed the following two problems.

1. Can each Lašnev space be embedded in a countably compact sequential regular space?

2. Is finite (or countable) product of Lašnev spaces subsequential?

This paper gives a negative answer to the first problem and a partial answer to the second as follows:

1. Any Lašnev space, which is not metrizable, cannot be embedded in any countably compact regular space with countable tightness.

2. Assuming the continuum hypothesis (CH), there exist regular Fréchet spaces X and Y such that $X\times Y$ is not subsequential.

Each Fréchet space is subsequential. Therefore the second result shows that even a finite product of subsequential spaces is not subsequential (cf. [8, p. 179]).

In this paper spaces are assumed to be T_1 and maps to be continuous onto.

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§ 2. Theorems.

DEFINITION 1 ([1, p. 954]). A space X has countable tightness if it has the following property: If $A\subset X$ and $x\in Cl_X A$, then $x\in Cl_X B$ for some countable $B\subset A$.

Let $R=\{0\}\cup\{1/n\,;n\in\omega_0\}$ be a convergent sequence. Let S be the disjoint union of a sequence $\{R(n)\,;n\in\omega_0\}$ of copies of R, let $A=\{0(n)\in R(n)\,;0(n)=0, n\in\omega_0\}$, and let $T=S/A$, the quotient space obtained from S by identifying A to a point q.