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On relations between conformal mappings and isomorphisms of spaces of analytic functions on Riemann surfaces

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§1. Introduction.

Let $\mathfrak{S}$ be the set consisting of all compact bordered Riemann surfaces. For $\bar{S}$ in $\mathfrak{S}$, we denote its interior and its border by S and ∂S, respectively. Let p $(\geqq 0)$ be the genus of $\bar{S}$ and q $(\geqq 1)$ be the number of boundary components of $\bar{S}$. We set

$$N=2p+q-1.$$

Furthermore we denote by $A(S)$ the set of all functions which are analytic in S and continuous on $\bar{S}$. It forms a Banach algebra with the supremum norm

$$\|f\|=\sup_{z\in S}|f(z)|.$$

For $\bar{S}$ and $\bar{S}'$ in $\mathfrak{S}$, let $L(A(S), A(S'))$ denote the set of all continuous invertible linear mappings of $A(S)$ onto $A(S')$. It is shown by Rochberg [4] that $L(A(S), A(S'))$ is nonvoid if S and S' are homeomorphic. We set

$$c(T)=\|T\|\,\|T^{-1}\|$$

for T in $L(A(S), A(S'))$. We have always

$$c(T)\geqq 1,$$

and we can easily see that $T/\|T\|$ is an isometry if and only if $c(T)=1$. If $T1=1$, then

$$1\leqq\|T\|\leqq c(T),\quad 1\leqq\|T^{-1}\|\leqq c(T).$$

Let z and z' be points of S and S', respectively. If there exist a positive number ε and an element T of $L(A(S), A(S'))$ such that

$$|f(z)-(Tf)(z')|\leqq\varepsilon\min(\|f\|, \|Tf\|)$$

for all f in $A(S)$, then we say that z and z' are *ε-related with respect to T*, or z and z' satisfy an *ε-relation with respect to T*.

The purpose of the present paper is to prove the following theorems: