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Metacompactness and subparacompactness of product spaces

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§ 1. Introduction.

The aim of this paper is to generalize Kramer [3, Theorems 4.3, 4.4, 4.9] which gives some sufficient conditions for the product space to be metacompact or subparacompact.

Alster and Engelking [1] constructed a subparacompact space X such that X^2 is not subparacompact. The product space S^2 of Sorgenfrey lines S is not metacompact, though S is metacompact. Thus subparacompactness and metacompactness do not have productive property. This is the case even if one factor is metrizable and the other factor is paracompact. Przymusiński [4] constructed a separable metric space M and a separable first countable Lindelöf regular (and hence paracompact) space Y such that $M\times Y$ is neither subparacompact nor metacompact.

We consider the subparacompactness and (countable) metacompactness of the product space $X\times Y$, where X is a P-space due to Morita [2, Definition 56.1] and Y is a Σ-space due to Nagami [2, Definition 57.1]. It is seen in [2, Theorem 57.14] that these notions are very effective to our consideration. This is why we restrict Y to the class of Σ-spaces.

In the sequel, all spaces are assumed to be T_1 and N to be the positive integers.

§ 2. Theorems.

DEFINITION 1. A space X is said to be a *Σ-space* if there exists a sequence $\{\mathcal{F}_n : n\in N\}$ of locally finite closed covers of X satisfying the following (Σ):

(Σ): If $p_n\in C(p, \mathcal{F}_n)=\cap\{F: p\in F\in\mathcal{F}_n\}$ for every $n\in N$, then $\{p_n : n\in N\}$ clusters in X.

Moreover, if for every point $p\in X$,

$$C(p)=\bigcap_n C(p, \mathcal{F}_n)$$