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Compact minimal submanifolds of a sphere with positive Ricci curvature

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1. Introduction.

Let M be an n-dimensional simply connected compact orientable submanifold minimally immersed in an $(n+p)$-dimensional sphere of constant curvature 1. The pinching problem with respect to the scalar curvature of M [6] [1] and the sectional curvature of M [4] [8] have been studied. In this note, we shall prove a pinching theorem with respect to the Ricci curvature of M. Some examples are:

EXAMPLE 1. In general, let $S^q(r)$ denote a q-dimensional sphere in R^{q+1} with radius r. Let m and n be positive integers such that $m<n$ and let $M_{m,n-m} = S^m\left(\sqrt{\frac{m}{n}}\right)\times S^{n-m}\left(\sqrt{\frac{n-m}{n}}\right)$. We imbed $M_{m,n-m}$ into $S^{n+1}=S^{n+1}(1)$ as follows. Let (u, v) be a point of $M_{m,n-m}$, where u (resp. v) is a vector in R^{m+1} (resp. R^{n-m+1}) of length $\sqrt{\frac{m}{n}}$ $\left(\text{resp. } \sqrt{\frac{n-m}{n}}\right)$. We can consider (u, v) as a unit vector in $R^{n+2}=R^{m+1}\times R^{n-m+1}$. It is easily shown that $M_{m,n-m}$ is a minimal submanifold of S^{n+1}. Furthermore from the fact the first eigenvalue of the Laplacian of $M_{m,n-m}$ is n and the dimension of the eigenspace is $n+2$, we can prove the following.

Let χ be a minimal immersion of $M_{m,n-m}$ into S^{n+p} such that the immersion is full, i.e. $\chi(M_{m,n-m})$ is not contained in a linear subspace of R^{n+p+1}. Then $p=1$ and the immersion is rigid. The Ricci curvature of $M_{m,n-m}$ varies between $\frac{n(m-1)}{m}$ and $\frac{n(n-m-1)}{n-m}$.

EXAMPLE 2. We can define a minimal immersion of an n-dimensional complex projective space $P^n_{2n/(n+1)}$ with holomorphic sectional curvature $\frac{2n}{n+1}$ into $S^{n(n+2)-1}$ such that the usual coordinate functions of $R^{n(n+2)}$ are all independent hermitian harmonic functions of degree 1 on $P^n_{2n/(n+1)}$.