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Injective envelopes of C^*-algebras

By Masamichi HAMANA

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§ 1. Introduction.

For a Banach space, the existence and uniqueness of its injective envelope was proved by Cohen [**5**], and the present author [**9**] generalized this result to the case of Banach modules over a unital Banach algebra. In this paper we show a C^*-algebraic version of these results, i.e. that any unital C^*-algebra has a unique injective envelope (Theorem 4.1), where injectivity for C^*-algebras is understood as that considered by several authors, e.g. Hakeda and Tomiyama [**8**], Tomiyama [**16**], Choi and Effros [**4**], Loebl [**12**], et al. We also give two characterizations of injective C^*-algebras, one of which (Proposition 4.8) is similar to that of injective Banach modules (cf. [**9**; Lemma 3 (iv)]) and another (Proposition 4.11) is similar to that of von Neumann algebras whose commutant has property P of Schwartz ([**13**]; cf. also Remark 4.13). In the last section we give an example of an injective non W^*-, AW^*-factor of type III.

We recall the above-mentioned result of Cohen [**5**]. He considered the category whose objects are Banach spaces and whose morphisms are contractive linear maps, and defined "injectivity" and an "injective envelope" of a Banach space as follows: A Banach space Y is injective if any continuous linear map of a linear subspace of a Banach space Z into Y extends to a continuous linear map of the same norm on all of Z. An injective envelope of a Banach space X is a pair (Y, κ) of an injective Banach space Y and a linear isometry κ of X into Y such that Y itself is the only subspace of Y which is injective and contains $\kappa(X)$ [or equivalently, the identity map id_Y on Y ($\mathrm{id}_Y(y)=y$, $y\in Y$) is a unique contractive linear map of Y into itself which fixes each element of $\kappa(X)$ (cf. Isbell [**10**])]. This pair (Y, κ) is unique in the sense that if (Y_1, κ_1) is another injective envelope of X, there exists a linear isometry ι of Y onto Y_1 such that $\iota\circ\kappa=\kappa_1$.

In contrast to the case of Banach spaces, we consider the category whose objects are unital C^*-algebras and whose morphisms are unit-preserving completely positive linear maps. Hereafter, unless otherwise specified, C^*-algebras