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Generalized Hasse-Witt invariants and unramified Galois extensions of an algebraic function field

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Introduction.

In this paper, we give a certain generalization of the Hasse-Witt theory (cf. [4]).

Let K be an algebraic function field with an algebraically closed constant field k of characteristic $p>0$, and g be its genus. Let M be the maximum unramified Galois extension of K. Let Δ_g be the group generated by $2g$ elements u_i, v_i $(i=1, \cdots, g)$ with the following fundamental relation:

$$(u_1 v_1 u_1^{-1} v_1^{-1}) \cdots (u_g v_g u_g^{-1} v_g^{-1})=1.$$

Let $\bar{\Delta}_g$ be the completion of Δ_g with respect to subgroups of finite index. Then, it is well known that there is a surjective homomorphism of $\bar{\Delta}_g$ onto $Gal(M/K)$, and that its kernel is contained in the intersection of kernels of continuous homomorphism from $\bar{\Delta}_g$ to finite groups with order prime to p. (cf. [3]).

It is obvious that the structure of $Gal(M/K)$ (as an abstract group) depends on g and p. We note that for any finite group G with order prime to p, the number of unramified Galois extensions of K whose Galois group is isomorphic to G is determined by g. Moreover, it is well-known that the structure of the Galois group of the maximal unramified abelian extension of K is determined by g, p, and the invariant γ_K that was introduced by Hasse-Witt (cf. [4]). Hence if $g=1$, $Gal(M/K)$ is determined by g, p, and γ_K. But if $g \geqq 2$, the structure of $Gal(M/K)$ is not determined only by g, p and γ_K.

In §1, we define an unramified D_{np^m}-extension of K as an unramified Galois extension of K whose Galois group is isomorphic to

$$D_{np^m}=\langle \sigma, \tau \mid \sigma^{p^m}=\tau^n=1,\ \tau\sigma\tau^{-1}=\sigma^i, \text{ where } i \text{ is a primitive } n\text{-th root of unity in } (\boldsymbol{Z}/p^m\boldsymbol{Z})^{\times} \rangle.$$