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Convolution powers of singular-symmetric measures

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1. Introduction.

Let G be a L.C.A. group and $\hat{G}$ be its dual group. Let $M(G)$ be the measure algebra on G and $L^1(G)$ be the group algebra on G. In [7], Taylor showed that: There are a compact topological abelian semigroup S and an isometric isomorphism θ of $M(G)$ into $M(S)$ such that;

(a) $\theta(M(G))$ is a weak-*dense subalgebra of $M(S)$;

(b) $\hat{S}$, the set of all continuous semicharacters on S, separates the points of S;

(c) for $f\in\hat{S}$, $\mu\rightarrow\int_S fd\theta\mu$ $(\mu\in M(G))$ is a non-zero complex homomorphism of $M(G)$;

(d) for a non-zero complex homomorphism F of $M(G)$, there is an $f\in\hat{S}$ such that $F(\mu)=\int_S fd\theta\mu$ for $\mu\in M(G)$.

We can consider that $\hat{S}$ is the maximal ideal space of $M(G)$, $\hat{G}\subset\hat{S}$, and the Gelfand transform of $\mu\in M(G)$ is given by $\hat{\mu}(f)=\int_S fd\theta\mu$ $(f\in\hat{S})$. A closed subspace (ideal, subalgebra) $N\subset M(G)$ is called an L-subspace (L-ideal, L-subalgebra) if $L^1(\mu)\subset N$ for every $\mu\in N$, where $L^1(\mu)=\{\lambda\in M(G);\ \lambda$ is absolutely continuous with respect to μ $(\lambda\ll\mu)\}$. We denote by $\mathrm{Rad}\,L^1(G)$ the radical of $L^1(G)$ in $M(G)$, that is, $\mathrm{Rad}\,L^1(G)=\{\mu\in M(G);\ \hat{\mu}(f)=0$, for all $f\in\hat{S}\backslash\hat{G}\}$. We put $\mathfrak{L}(G)=\sum_{\tau}\mathrm{Rad}\,L^1(G_\tau)$, where τ runs through over L.C.A. group topologies on G which are stronger than the original one. Then $\mathfrak{L}(G)\subset M(G)$ and $\mathfrak{L}(G)$ is an L-subalgebra ([2]). For $\mu\in M(G)$, we put $\mu^*(E)=\overline{\mu(-E)}$ for every Borel subset E of G. We denote by $\mathfrak{M}$ the set of all symmetric measures of $M(G)$, that is, $\mathfrak{M}=\{\mu\in M(G);\ \hat{\mu}^*(f)=\overline{\hat{\mu}(f)}$ for every $f\in\hat{S}\}$. Then it is easy to show that $\mathfrak{L}(G)\subset\mathfrak{M}$. A measure $\mu\in\mathfrak{M}$ is called singular-symmetric if μ is singular with