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On a class of type I solvable Lie groups I

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1. Introduction.

Let G be a topological group, and U a unitary representation of G. If the von Neumann algebra generated by $\{U_x ; x\in G\}$ is of type I, we say that the representation is of type I. The group G is said to be of type I if all the strongly continuous unitary representations of G are of type I.

In this paper, we shall restrict ourselves to connected, simply connected solvable Lie groups. Then such a G is known to be of type I either if G is an exponential group by O. Takenouchi [**10**], or if G is the universal covering group of the identity component of a suitable algebraic group in $GL(m, \boldsymbol{R})$, by results of J. Dixmier [**2**] and L. Pukanszky [**9**].

Let G be a connected Lie subgroup of $GL(m, \boldsymbol{R})$. In a previous paper, the author defined G to be *semi-algebraic* if a maximal compact connected subgroup of the algebraic hull of G is contained in G. The purpose of this paper is to prove the following theorem:

THEOREM. *Let G be a connected, simply connected solvable Lie group. If the adjoint group $Ad(G)$ of G is semi-algebraic, then G is of type I.*

Let G be a Lie group, and let g be the Lie algebra of G. Let g^* denote the vector space dual to g. We define a map $\mu : G\ni x \mapsto \mu(x) \in GL(g^*)$ by

$$\langle\mu(x)f, X\rangle=\langle f, Ad(x^{-1})X\rangle \quad \text{for} \quad X\in g \quad \text{and} \quad f\in g^*.$$

Then μ is a representation (the *coadjoint representation*) of G, and the corresponding representation $d\mu$ of the Lie algebra g is given by

$$\langle d\mu(X), Y\rangle=\langle f, [Y, X]\rangle,$$

for $X, Y\in g$ and $f\in g^*$. For $f\in g^*$, we put

$$G(f)=\{x\in G ; \mu(x)f=f\},$$

$$g(f)=\{X\in g ; d\mu(X)f=0\}.$$

Then $G(f)$ is the isotropy group at f, and $g(f)$ is the Lie algebra of $G(f)$.