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A difference inequality and its application to nonlinear evolution equations

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Introduction.

Let H be a real Hilbert space and V, W be real Banch spaces with $V\subset W\subset H$. We assume V is dense in W and H, and the natural injections from V into W and from W into H are continuous. We identify H with its dual H^* (i.e., $V\subset W\subset H\subset W^*\subset V^*$). Let us consider the nonlinear evolution equations

$$u''(t)+B(t)u'(t)+A(t)u(t)=f(t) \tag{a}$$

and

$$B(t)u'(t)+A(t)u(t)=f(t) \tag{b}$$

where $A(t)$ is the Fréchet derivative of a functional $F_{A(t)}(u)$ on V and B is a bounded operator from W to W^*.

Recently in [6], the author has discussed the decay property of solutions of the equations (a) and (b) in the case $A(t)$ and $B(t)$ are independent of t. There the problem is reduced to the difference inequality of the form

$$\sup_{s\in[t,t+1]}\phi(s)^{1+r}\leqq C(\phi(t)-\phi(t+1))+\delta(t) \qquad (r\geqq 0)$$

where C is a positive constant, $\phi(t)$ is a nonnegative function on $R^+=[0, \infty)$ and $\delta(t)$ is a function tending to 0 as $t\to\infty$. The decay property of $\phi(t)$ as $t\to\infty$ has been discussed in [4] and [5] with applications to the wave equation with a nonlinear dissipative term.

In this paper we first treat a more general difference inequality

$$\sup_{s\in[t,t+1]}\phi(s)^{1+r}\leqq C(1+t)^{\alpha}(\phi(t)-\phi(t+1))+\delta(t)$$

where α, r are constant with $0\leqq\alpha\leqq 1$, $r\geqq 0$. Next, the result for the above inequality is applied to the investigation of the asymptotic behaviour of the solutions of (a) and (b). As is in [6], the equation (a) will be treated in detail, while we give a brief discussion of eq. (b), because the latter is simpler.

A typical example which our result is applicable to is the nonlinear generalized Euler-Poisson-Darboux equation