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On the cubics defining abelian varieties

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Introduction.

Let k be an algebraically closed field of characteristic p, X an abelian variety over k of dimension g, and L an ample invertible sheaf on X. For any integer $a \geqq 3$, we denote by $\phi_a: X \to \boldsymbol{P}(\Gamma(L^a))$ the canonical embedding of X. The purpose of the present paper is to prove, except the case of $p=2$ and 3, the statement:

$\phi_3(X)$ *is ideal-theoretically an intersection of cubics.*

For generic polarized abelian varieties, the statement is proved by Morikawa [4] for any characteristic, using deformations of polarized abelian varieties. For $a \geqq 4$, Mumford ([5], Theorem 10) proved that for any characteristic, $\phi_a(X)$ is ideal-theoretically an intersection of quadrics. We shall prove our assertion stated above, by reducing it to Mumford's theorem. The essential tool in the reduction process is the normal generation of $\phi_3(X)$, which is discovered by Koizumi [2] for characteristic zero, and later generalized by the author [7], [8] for any characteristic.

Section 1 is devoted to recalling some results concerning the normal generation of abelian varieties. In Section 2, we shall give a slight modification of Mumford's theorem, in order that it will be fit for later use. The proof of our result will be completed in Section 3.

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NOTATION. Throughout the paper, k is an algebraically closed field of characteristic p, and X is an abelian variety over k of dimension g. We denote by $\hat{X}$ the dual abelian variety of X, and by P the Poincaré invertible sheaf on $X \times \hat{X}$. For any $\hat{x} \in \hat{X}$, we put $P_{\hat{x}} = P|_{X \times \{\hat{x}\}}$. For any integer n, we put $X_n = \{x \in X \mid nx = 0\}$. For an invertible sheaf L on X, we abbreviate $\Gamma(X, L)$ by $\Gamma(L)$, and we denote

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