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On the invariant for a certain type of involutions on homology 3-spheres and its application

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§ 1. Introduction.

The purpose of this paper is first to define a Z-invariant for involutions on homology 3-spheres which have circles as the sets of fixed points. Our definition of the invariant is derived from Hirzebruch's formula about the signature of ramified coverings [2]. For the second we shall present examples of involutions which are distinguished by our invariant. Finally, as an application of this invariant, we shall show a theorem on 4-dimensional homotopy smoothings. It will be proved that $\mathcal{HS}(P^2\times D^2, \partial)$ has a nontrivial element.

§ 2. Definition of $\sigma(H, \tau)$.

We shall work in the smooth category. The following notations and conventions are used throughout the paper.

For an involution $T: X\rightarrow X$, X/T denotes its orbit space, and Fix$T$ denotes the set of fixed points of T. When $A\subset X$ is invariant under T, we write A/T instead of $A/(T|A)$. Let $i: X^n\rightarrow Y^m$ be an embedding of a compact oriented n-manifold X into an m-manifold Y such that $i^{-1}(\partial Y)=\partial X$. Then $[X, \partial X]$ denotes a homology class in $H_n(Y, \partial Y)$ represented by $(X, \partial X)$. As usual, $H_n(Y, \partial Y)$ means an n-dimensional integral homology group of $(Y, \partial Y)$. For a homology class x, $x^2\in Z$ denotes its self-intersection number whenever it is defined. Suppose that a manifold X and its boundary A are oriented, we write $\partial X=A$ when {the orientation of A} $\times$ {the outward normal vector} coincides with the orientation of X.

Now we define $\sigma(H, \tau)$. Let H^3 be a homology 3-sphere, that is, a closed 3-manifold having an integral homology group isomorphic to that of a 3-sphere. Let τ be a smooth involution on H whose fixed points set Fixτ is diffeomorphic to a circle S^1. For (H, τ), we define the signature $\sigma(H, \tau)\in Z$ using the following two lemmas.

LEMMA 1. *H/τ is a homology 3-sphere.*