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On a mixed problem for $\square$ with a discontinuous boundary condition (II)
—an example of moving boundary—

By Atsushi INOUE

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§ 1. Introduction.

Let Ω be a domain in $\boldsymbol{R}^n$ with smooth *compact*[1] boundary $\partial\Omega$. That is, Ω is the interior or exterior domain of $\partial\Omega$.

Consider the following Initial-Boundary-Value Problem (in short, I. B. V. P. or mixed problem).

$$(1.1)\quad \begin{cases} \square u(x, t)=f(x, t) & \text{in} \quad Q=\Omega\times(0, T), \\ u(x, 0)=u_0(x), & \\ \dfrac{\partial u}{\partial t}(x, 0)=u_1(x), & \end{cases}$$

$$(1.2)\quad \begin{cases} u(\tilde{x}, t)^{2)}=0 & \text{on} \quad \Sigma_D=\bigcup_{0\leq t\leq T}\partial_D\Omega(t)\times\{t\} \quad \text{and} \\ \dfrac{\partial u}{\partial \boldsymbol{\nu}}(\tilde{x}, t)=0 & \text{on} \quad \Sigma_N=\bigcup_{0\leq t\leq T}\partial_N\Omega(t)\times\{t\} \end{cases}$$

where $\square=\dfrac{\partial^2}{\partial t^2}-\sum_{j=1}^{n}\dfrac{\partial^2}{\partial x_j^2}$, $\boldsymbol{\nu}$: the unit exterior normal of $\partial\Omega$ and $\partial_D\Omega(t)$ and $\partial_N\Omega(t)$ are open sets in $\partial\Omega$ for each $t\in[0, T]$ which change with t and satisfy $\partial_D\Omega(t)\cap\partial_N\Omega(t)=\emptyset$, $\partial_D\Omega(t)\cup\overline{\partial_N\Omega(t)}=\overline{\partial_D\Omega(t)}\cup\partial_N\Omega(t)=\partial\Omega$, $\Gamma(t)=\overline{\partial_D\Omega(t)}\cap\overline{\partial_N\Omega(t)}$: $(n-2)$ dimensional smooth manifold in $\boldsymbol{R}^n$. We write $\Gamma=\bigcup_{0\leq t\leq T}\Gamma(t)\times\{t\}$.

For the future use, we rewrite the boundary condition (1.2) in the following form.

$$(1.2)'\qquad Y(\tilde{x}, t)\frac{\partial u}{\partial \boldsymbol{\nu}}(\tilde{x}, t)+(1-Y(\tilde{x}, t))u(\tilde{x}, t)=0 \quad \text{on} \quad \Sigma=\partial\Omega\times[0, T],$$

1) This is assumed only for the sake of simplicity. If $\partial\Omega$ is not compact, we must add some uniformity assumption at ∞ in the following argument.

2) We represent the generic points in Ω and $\partial\Omega$ by x and $\tilde{x}$, respectively.