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On representations of finite groups over skewfields

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Let G be a finite group and D a skewfield. A D-representation of the group G is a homomorphism of G into $GL(n, D)$ where $GL(n, D)$ is the group of all nonsingular $n\times n$ matrices over D. Equivalence, irreducibility, etc. of such representations are defined in the usual manner.

The following question arises:
What is the number of equivalence classes of irreducible D-representations of G? The answer to this question for the case when D is a skewfield of real quaternions was given by J. E. Houle [4] who showed that if r and r' are respectively the number of conjugacy classes and the number of selfinverse conjugacy classes of a finite group G, then the number of equivalence classes of irreducible representations of G over the real quaternions is equal to $\frac{r+r'}{2}$. The aim of this note is to find the group theoretical characterisation of the number of equivalence classes of irreducible D-representations of a finite group where D is finite dimensional over its centre.

1. Notation and definitions. D is a skewfield with characteristic $p\geqq 0$. K is the centre of D. D_n is the ring of all $n\times n$ matrices over D. $\overset{K}{\sim}$ (respectively $\overset{D}{\sim}$) is the K-equivalence (respectively D-equivalence). Let A and B be K-algebras. We shall call two (A, B)-modules M_1 and M_2 isomorphic if and only if M_1 and M_2 are isomorphic regarded as left A-modules and right B-modules.

Finally, let n be the least common multiple of the orders of the p'-elements in G and let ε be a primitive n-th root of unity over K. Let I_n be the multiplicative group consisting of those integers r, taken modulo n, for which $\varepsilon\rightarrow\varepsilon^r$ defines an automorphism of $K(\varepsilon)$ over K. Two p'-elements $a, b, \in G$ are called K-conjugate if $x^{-1}bx=a^r$ for some $x\in G$ and some $r\in I_n$.

2. The number of equivalence classes of irreducible representations of a finite group over a skewfield. If D is a field we may treat the terms matrix representation and DG-module as interchangeable. Slight modification is needed for the case when D is a skewfield. Namely, the following lemma holds.