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On a duality for C^*-crossed products by a locally compact group

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Abstract.

Let $(\mathfrak{A}, G, \alpha)$ be a C^*-dynamical system, and $C_r^*(\mathfrak{A}; \alpha)$ the reduced C^*-crossed product of $\mathfrak{A}$ by α. We construct a "dual" C^*-crossed product $C_d^*(C_r^*(\mathfrak{A}; \alpha); \beta)$ of $C_r^*(\mathfrak{A}; \alpha)$ by an isomorphism β from $C_r^*(\mathfrak{A}; \alpha)$ into the full operator algebra $\mathcal{L}(\mathfrak{K})$ on a Hilbert space $\mathfrak{K}$. Then, it is isomorphic to the C^*-tensor product $\mathfrak{A}\hat{\otimes}_*\mathcal{C}(L^2(G))$ of $\mathfrak{A}$ and the C^*-algebra $\mathcal{C}(L^2(G))$ of all compact operators on $L^2(G)$.

In the abelian case, there exists a continuous action $\hat{\alpha}$ of the dual group $\hat{G}$ of G on the C^*-crossed product $C^*(\mathfrak{A}; \alpha)$ of $\mathfrak{A}$ by α such that the C^*-crossed $C^*(C^*(\mathfrak{A}; \alpha); \hat{\alpha})$ of $C^*(\mathfrak{A}; \alpha)$ by $\hat{\alpha}$ is isomorphic to $C_d^*(C_r^*(\mathfrak{A}; \alpha); \beta)$.

§ 1. Introduction.

In [4], the second author showed a C^*-algebra version of Takesaki's duality theorem for crossed products of von Neumann algebras. In other words, given a C^*-dynamical system $(\mathfrak{A}, G, \alpha)$ based on a locally compact abelian group G, there exists a continuous action $\hat{\alpha}$ of the dual group $\hat{G}$ of G on the C^*-crossed product $C^*(\mathfrak{A}, \alpha)$ of $\mathfrak{A}$ by α such that the C^*-dynamical system $(C^*(C^*(\mathfrak{A}; \alpha); \hat{\alpha}), G, \hat{\hat{\alpha}})$ is equivalent to the C^*-dynamical system $(\mathfrak{A}\hat{\otimes}_*\mathcal{C}(L^2(G)), G, \alpha\otimes \mathrm{Ad}(\lambda))$, where $\mathcal{C}(L^2(G))$ is the C^*-algebra of all compact operators on $L^2(G)$, and λ is the regular representation of G on $L^2(G)$.

Recently, Y. Nakagami [3] generalized Takesaki's duality theorem based on abelian groups to non-abelian groups using the method on Hopf-von Neumann algebras. (Also see [2].)

In this paper, we study a non-abelian duality for C^*-crossed products referring to Nakagami's construction in von Neumann algebras. Actually, we obtain that for a C^*-dynamical system $(\mathfrak{A}, G, \alpha)$, there exists an isomorphism β of the reduced C^*-crossed product $C_r^*(\mathfrak{A}; \alpha)$ of $\mathfrak{A}$ by α into the full operator algebra $\mathfrak{L}(L^2(G\times G; \mathfrak{H}))$ on the Hilbert space $L^2(G\times G; \mathfrak{H})$ such that the "dual" C^*-crossed product $C_d^*(C_r^*(\mathfrak{A}; \alpha); \beta)$ is isomorphic to the tensor product $\mathfrak{A}\hat{\otimes}_*\mathcal{C}(L^2(G))$.