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The maximal ideal space of certain algebra $H^{\infty}(m)$

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§1. Introduction.

Let A be a uniform algebra on a compact Hausdorff space X and m a complex homomorphism of A. We suppose that m has a unique representing measure μ_m on X and that the Gleason part $P(m)$ containing m consists of more than one point. We denote by $H^{\infty}(m)$ the w^* closure of A in $L^{\infty}(d\mu_m)$, and by I^{∞} the ideal $\{f\in H^{\infty}(m): \varphi(f)=0$ for all $\varphi\in P(m)\}$ of $H^{\infty}(m)$. In [**10**], Merrill proved that $H^{\infty}(m)$ is maximal as a w^* closed subalgebra of $L^{\infty}(d\mu_m)$ if and only if $I^{\infty}=\{0\}$. In this paper we shall deal with the case when $I^{\infty}\neq\{0\}$.

In §2 we shall state some preliminaries and two lemmas. In §3 we shall study some properties of the maximal ideal space of the Banach algebra $H^{\infty}(m)$ with $I^{\infty}\neq\{0\}$. In §4 we shall study some properties of a Gleason part $P(m)$ such that $A|P(m)=H^{\infty}(D)$ (for the precise meaning see §4). In §5 we shall give some examples relating to §3 and §4.

§2. Preliminaries and lemmas.

For a complex commutative Banach algebra B, let B^{-1} be the set of all invertible elements of B. Let $M(B)$ be the maximal ideal space of B endowed with the Gelfand topology, let $\hat{f}$ and $\hat{B}$ be the Gelfand transforms of f $(\in B)$ and B respectively, and let $\Gamma(B)$ be the Šilov boundary of B.

Let X be a compact Hausdorff space, and let $C(X)$ $(C_R(X))$ be the complex (real) Banach algebra of all complex (real) valued continuous functions on X. Let A be a *uniform algebra* on X, i.e., A is a uniformly closed subalgebra of $C(X)$ which contains the function 1 and separates the points of X. A representing measure for $\varphi\in M(A)$ is a probability measure μ on X such that $\varphi(f)=\int f d\mu$ for all $f\in A$. We denote by $\operatorname{supp}\mu$ the closed support of a measure μ. When $\varphi\in M(A)$ has a unique representing measure, sometimes we use the same symbol φ to denote its representing measure. Given φ and ψ in $M(A)$, we set

$$d(\varphi, \psi)=\sup\{|\varphi(f)| : f\in A,\ \|f\|=\sup|f|\leqq 1,\ \psi(f)=0\}$$