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A variation of Lyndon-Keisler's homomorphism theorem and its applications to interpolation theorems

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As is well known, R. C. Lyndon [9] proved the preservation theorem for homomorphisms and some related theorems by using his interpolation theorem (cf. [8]). H. J. Keisler [4] gave a simple proof of a generalization of the essential part of the above theorems of Lyndon on homomorphisms by using his theory of generalized atomic sets of formulas. On the other hand, L. Henkin [3] proved an extended form of the Craig-Lyndon interpolation theorem which includes the completeness theorem. Improving Henkin's proof, A. Oberschelp [12] proved an interpolation theorem of Lyndon type whose interpolant has some information about the equality symbol. Before this, the Craig-Lyndon interpolation theorem for the infinitary language $L_{\omega_1\omega}$ was given by E.G.K. Lopez-Escobar [7].

The main purpose of this paper is to prove some interpolation theorems whose interpolants have more information about the equality symbol and non-logical symbols than those of the above well-known interpolation theorems, by using a new notion of morphisms.

In Section 1, we shall introduce the notion of a morphism which can be naturally obtained from the notion of a homomorphism by using a many-to-many correspondence instead of a mapping. As preparation for the next section, we shall state a theorem on morphisms which is an immediate variation of Lyndon-Keisler's homomorphism theorem (cf. Lyndon [**9**; p. 151, lines 3-6], Keisler [**4**; Theorem 3]).

In Section 2, we shall prove the following interpolation theorem, which may be regarded as a strengthened version of Craig's, of Lyndon's, and of Oberschelp's (cf. Craig [**2**; Theorem 5], Lyndon [**8**; p. 140], Oberschelp [**12**; Theorem 2], and also Chang and Keisler [**1**; Theorems 2.2.20 and 2.2.24], Robinson [**14**; Theorem 5.1.8], Shoenfield [**15**; p. 80]).

Let Φ and Ψ be sentences of a first order language with the sentential constants $\mathfrak{t}$ *and* $\mathfrak{f}$ *and with or without equality such that $\Phi \models \Psi$. Then there exists a sentence Θ such that* (1) *$\Phi \models \Theta$ and $\Theta \models \Psi$,* (2) *all relation symbols occurring positively* (*resp. negatively*) *in Θ occur positively* (*resp. negatively*) *in*