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On transitive groups in which the maximal number of fixed points of involutions is five

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§ 1. Introduction.

Let t and μ be integers such that $t\geqq 1$, $\mu\geqq 0$. A finite permutation group (G, Ω) of even order is said to be a (t, μ)-group if G is t-transitive on Ω and μ is the maximal number of the fixed points of involutions in G. All $(2, \mu)$-groups with $\mu\leqq 4$ have been classified; for $\mu=0$ and $\mu=1$ by Bender [**2**][**3**], for $\mu=2$ by Hering [**12**], for $\mu=3$ by King [**14**] and for $\mu=4$ by Noda [**15**] and Buekenhout [**4**]. The (1, 3)-groups have been classified by Buekenhout [**5**] and (1, 4)-groups have been studied by Rowlinson and Buekenhout [**6**][**20**]. In [**18**][**19**], Rowlinson has shown that a simple $(1, \mu)$-group with one conjugate class of involutions is one of the known simple groups when $1\leqq\mu\leqq 7$.

In this paper we shall consider primitive (1, 5)-groups. Let $(\tilde{G}, \Omega)$ be a primitive (1, 5)-group and G be a minimal normal subgroup of $\tilde{G}$.

If G is solvable, G is an elementary abelian p-group for some prime p. In this case we can easily show that $p=5$. Moreover $\tilde{G}$ is a group of automorphisms of an affine space satisfying one of the following:

(1) Dimension of the affine space is 2 or 3.

(2) If T is a Sylow 2-subgroup of $\tilde{G}_\alpha$ $(\alpha\in\Omega)$ then T is cyclic or generalized quaternion and $|C_G(z)|=5$ where z is a unique involution in T.

If G is not solvable, G is a direct product of r isomorphic nonabelian simple groups. In this case, the permutation group (G, Ω) is a $(1, \mu)$-group where $\mu\in\{1, 3, 5\}$ and we can easily show that $r=1$, with the exception of the following case

$$G=G_1\times G_2\cong A_5\times A_5$$

where G_i $(1\leqq i\leqq 2)$ is isomorphic to the alternating group of degree 5 and G is a permutation group on the set $\{(i, j)\,|\,1\leqq i, j\leqq 5\}$, which is defined by $(i, j)^g=(i^{g_1}, j^{g_2})$ for $g=g_1\cdot g_2\in G$ with $g_i\in G_i$ $(1\leqq i\leqq 2)$. Thus we have $\mathrm{Aut}(G)\geqq\tilde{G}\geqq G$, where G is a simple $(1, \mu)$-group $(\mu\in\{1, 3, 5\})$ or the group isomorphic to $A_5\times A_5$. Since simple (1, 1)-groups and (1, 3)-groups are known simple groups by Bender [**3**], Buekenhout [**5**] and Rowlinson [**18**], we may consider simple (1, 5)-