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Fine movability

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1. Introduction.

The notion of shape for compacta was introduced by K. Borsuk [4]. In a series of papers [5], [6] and [7] Borsuk has defined the concepts of fundamental absolute neighborhood retracts (FANR's), movability and n-movability and has proved that all of these properties are shape invariants.

In this paper we shall introduce the concepts of fine movability and n-fine movability, $n=0, 1, 2, \cdots$, which are shape invariants and define the n-fine movability pro-group $\boldsymbol{m}_n(X, x_0)$ for a pointed compactum (X, x_0). For a compactum X, we shall prove that

(1) X is a pointed FANR, i. e. an FANR having the shape of a CW-complex if and only if X is fine movable,

(2) X is n-fine movable if and only if $\boldsymbol{m}_k(X, x_0)=0$ for $x_0 \in X$ and $k=0, 1, 2, \cdots, n$,

(3) an n-dimensional compactum X is fine movable if and only if $\boldsymbol{m}_k(X, x_0)=0$ for $x_0 \in X$ and $k=0, 1, 2, \cdots, n+1$,

(4) an LC^{n-1} compactum X is n-fine movable,

(5) if X is n-fine movable X is n-movable,

(6) if X_1, X_2 and $X_1 \cap X_2$ are n-fine movable compacta $X_1 \cup X_2$ is n-fine movable.

From (1), (2) and (5) we have the following implications: a pointed FANR $\rightarrow$ an n-fine movable compactum $\rightarrow$ an n-movable compactum. It is known that each of converse implications does not generally hold. S. Mardešić [18] has proved that an n-dimensional LC^{n-1} compactum is movable. The assertions (4) and (5) extend this result. For pointed FANR's or equivalently for fine movable compacta (6) has proved by Dydak, Nowak and Strok [12].

Throughout this paper all spaces are metrizable and maps are continuous. AR and ANR mean those for metric spaces. By $\dim X$ we mean the covering dimension of X.

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