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Perturbations of M-accretive operators and quasi-linear evolution equations

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1. Introduction.

Let X be a complex Banach space and let A be an m-accretive operator with domain $D(A)\subseteqq X$ and range $R(A)\subseteqq X$. Then a rather common problem in nonlinear perturbation theory is the following: given an accretive operator $B: D(B)\rightarrow X(D(B)\subseteqq D(A))$, what additional assumptions on B ensure the m-accretiveness of $A+B$? This problem can be rephrased as follows: let $U(v)u=Au+Bv$, $(u, v)\in D(A)\times D(A)$. Assume that U is m-accretive in u and accretive in v. What additional assumptions on U w.r.t. v ensure the m-accretiveness of the operator $U_1: u\rightarrow U(u)u$? Our main purpose here is to present such a result for operators $U(v)u$ which are not necessarily equal to the sum of two operators A and B as above. This result will be shown after we establish the existence of solutions to quasi-linear problems of the form

$$\text{(I)} \qquad x'(t)+U(x(t))x(t)=0\,, \qquad x(0)=x_0\,, \qquad t\in[0, \infty)\,.$$

The method here employs the contraction principle on an operator T associated with the equation

$$\text{(II)}_u \qquad x'(t)+U(u(t))x(t)=0\,, \quad x(0)=x_0\,, \quad t\in[0, T]\,,$$

where u is taken from a suitable family of continuous functions. This operator T maps $u(t)$ into the unique solution $x_u(t), t\in[0, T]$ of $(\text{II})_u$ which is assumed to exist by known results. In case $U(v)u$ is linear in u, the problem $(\text{II})_u$ is linear, and this is why problems like (I) are called "quasi-linear".

Quasi-linear problems for ordinary differential equations go at least as far back as Corduneanu [**1**]. The reader is also referred to the papers of Lasota and Opial [**11**], Opial [**15**], Avramescu [**2**], Kartsatos [**4-6**] and Kartsatos and Ward [**7**] for some further results. For quasi-linear problems concerning partial differential equations, the reader is referred to Kato [**10**], Ward [**16**] and the references therein. Ward employed in [**16**] the Schauder-Tychonov theorem for a suitable space of functions associated with the weak