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Non-singular bilinear maps which come from some positively filtered rings

By Yôichi MIYASHITA

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Let K be a commutative ring, and $K[X]$ the polynomial ring over K. Then it is known that $K[X]/(f(X))$ is a free Frobenius extension of K (in the sense of [3]) for a monic polynomial $f(X)$ ([5], [2]). The purpose of this paper is to extend this result to non-commutative rings. To this end we take a "positively filtered ring" satisfying some condition in place of a "polynomial ring" $K[X]$, and an ideal generated by a monic polynomial is replaced by a one sided ideal generated by a monic submodule, which is a generalization of a monic polynomial. Main results are Theorem 9, 11, and 12. In particular, Theorem 12 yields that $K[X]$ is a free Frobenius extension of $K[f(X)]$ for a monic polynomial $f(X)$ over a commutative ring K, and Corollary to Theorem 12 is a generalization of [5; Theorem 2.1].

§ 1.

All rings are associative, but not necessarily commutative. Every ring has 1, which is preserved by homomorphisms, inherited by subrings and acts as the identity operator on modules. Let ${}_AM$, ${}_AN$ be left A-modules over a ring A. By $\mathrm{Hom}_r({}_AM, {}_AN)$ we denote the module of left A-homomorphisms from ${}_AM$ to ${}_AN$ acting on the right side. We denote $\mathrm{Hom}_r({}_AM, {}_AM)$ by $\mathrm{End}_r({}_AM)$. Similarly Hom_l is used for right A-modules and right A-homomorphisms acting on the left side. Let ${}_AM_{A'}$ be a left A, right A'-module. If ${}_AM$ is finitely generated, projective, and generator, and $\mathrm{End}_r({}_AM)\simeq A'$ under the mapping induced by $M_{A'}$, we call ${}_AM_{A'}$ an invertible module. It is well known that this is right-left symmetric.

Let $R\supseteqq K$ be rings, and $R_0=K\subseteqq R_1\subseteqq R_2\subseteqq \cdots$ an ascending sequence of additive subgroups such that $R=\cup R_i$ and $R_i\cdot R_j\subseteqq R_{i+j}$ for all $i, j\geqq 0$. We call $R=\cup R_i$ a positively filtered ring over K. If, further, $R=\cup R_i$ satisfies the following condition we call $R=\cup R_i$ a (*)-positively filtered ring over K:

(*) Each R_n/R_{n-1} $(n\geqq 1)$ is an invertible module as a K-bimodule, and $(R_n/R_{n-1})\otimes_K(R_m/R_{m-1})\simeq R_{n+m}/R_{n+m-1}$ canonically, for all $n, m\geqq 1$.

We denote this by $K[R_1]$, and put $R_i=0$, if $i<0$. For any $i\geqq 0$, we put $gr_iR=R_i/R_{i-1}$. It is easily seen that the latter half of (*) can be replaced by