J. Math. Soc. Japan
Vol. 29, No. 4, 1977

On Čech homology and a stability theorem in shape theory

By Tadashi WATANABE

(Received Feb. 16, 1976)
(Revised May 31, 1976)

§ 1. Introduction and theorems.

The notion of shape was originally introduced by Borsuk and has been extended by many authors (see Borsuk [2] and [3] for shape theory). In this paper we shall use the ANR-systems approach to shape theory of Mardešić and Segal [9].

In [1], Borsuk introduced the notion of movability and raised the following problem: Let X and A be movable compact metric spaces such that $A\subset X$. Is the Čech homology sequence of such a pair (X, A) necessarily exact? Concerning this problem, Overton [11] constructed a counter-example and proved that Borsuk's problem is true if a pair (X, A) is movable. In this paper we shall consider Borsuk's problem under a different condition. Specifically we show the following theorem.

THEOREM 1. *Let X and A be movable compact metric spaces such that $A\subset X$. If the n-th Čech homology of X is a countable group for each n, then the Čech homology sequence of (X, A) is exact.*

Recently Edward and Geoghegan [6] proved a very important theorem, "stability theorem", which gives algebraic characterizations of FANR-continua in terms of the category of pro-groups or by using topologies on shape groups. These characterizations, however, are not simple. Therefore, in this paper we shall give some improvements on these characterizations. That is, we shall show the following theorem.

THEOREM 2. *Let (X, x) be a pointed compact connected metric space with finite dimension. Then the following conditions are equivalent:*

(A) *(X, x) is pointed movable and the n-th shape group of (X, x) is a countable group for each $n\geqq 1$.*

(B) *The n-th pro-homotopy groups of (X, x) satisfies Mittag-Leffler condition and the n-th shape group of (X, x) is a countable group for each $n\geqq 1$.*

(C) *(X, x) is a pointed FANR-space.*