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Nonlinear oscillation of second order functional differential equations with advanced argument

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§ 1. Introduction.

In this paper we consider the nonlinear second order functional differential equation with advanced argument

$$[r(t)y'(t)]'+f(y(g(t)), t)=0\,. \tag{1}$$

The conditions we always assume for r, g, f are as follows:

(a) $r(t)$ is continuous and positive for $t\geqq\alpha$;
(b) $g(t)$ is continuous for $t\geqq\alpha$, and $g(t)\geqq t$;
(c) $f(y, t)$ is continuous for $|y|<\infty$, $t\geqq\alpha$, and $yf(y, t)>0$ for $y\neq 0$, $t\geqq\alpha$.

It is convenient to classify equations of the form (1) according to the nonlinearity of $f(y, t)$ with respect to y. Equation (1) is called *superlinear* if, for each fixed t, $f(y, t)/y$ is nondecreasing in y for $y>0$ and nonincreasing in y for $y<0$. It is called *strongly superlinear* if there exists a number $\sigma>1$ such that, for each fixed t, $f(y, t)/|y|^{\sigma}\operatorname{sgn} y$ is nondecreasing in y for $y>0$ and nonincreasing in y for $y<0$. Equation (1) is called *sublinear* if, for each fixed t, $f(y, t)/y$ is nonincreasing in y for $y>0$ and nondecreasing in y for $y<0$. It is called *strongly sublinear* if there exists a number $\tau<1$ such that, for each t, $f(y, t)/|y|^{\tau}\operatorname{sgn} y$ is nonincreasing in y for $y>0$ and nondecreasing in y for $y<0$. This classification includes the corresponding classification of the equations of the form

$$[r(t)y'(t)]'+y(g(t))F([y(g(t))]^{2}, t)=0 \tag{2}$$

as given in [**6**]. (See also [**5**] and [**9**].)

In what follows we restrict our discussion to those solutions $y(t)$ of (1) which exist on some ray $[T_{y}, \infty)$ and satisfy $\sup\{|y(t)| : t\geqq T\}>0$ for every $T\geqq T_{y}$. Such a solution is said to be *oscillatory* if the set of its zeros is not bounded; otherwise, it is said to be *nonoscillatory*. Equation (1) itself is called oscillatory if all of its solutions are oscillatory.