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On the irreducible characters of the finite unitary groups

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Introduction.

Let k be a finite field of q elements, and k_2 the quadratic extension of k. Let σ be the automorphism of the finite general linear group $GL_n(k_2)$ defined by

$$(x_{ij})^{\sigma}=(x_{ji}^{q})^{-1}$$

for any element $(x_{ij})_{1\leqq i,j\leqq n}$ of $GL_n(k_2)$. The group $U_n(k_2)$ of σ-fixed elements of $GL_n(k_2)$ is called the finite unitary group over k_2. So far, the irreducible complex characters of $U_n(k_2)$ have been determined only for small n (see Ernola [4] and Nozawa [8], [9]), while those of $GL_n(k_2)$ have been determined completely by J. A. Green [7]. The purpose of the present paper is to give a method by which one can construct the irreducible complex characters of $U_n(k_2)$ using those of $GL_n(k_2)$, at least if the characteristic of k is not 2. As an application, we also obtain a parametrization of the irreducible characters of $U_n(k_2)$ which is dual to a known parametrization of the conjugacy classes.

Let χ be an irreducible character of $GL_n(k_2)$ which is fixed by σ, i. e. satisfies $\chi(x)=\chi(x^{\sigma})$ for all $x\in GL_n(k_2)$. Then, by a well-known elementary lemma, χ can be extended to an irreducible character $\tilde{\chi}$ of the semi-direct product $AGL_n(k_2)$ of $GL(k_2)$ with the group $A=\{1, \sigma\}$. Our main theorem is:

Assume that char $(k)\neq 2$. *Let* χ *be a* σ*-fixed irreducible character of* $GL_n(k_2)$, *and* $\tilde{\chi}$ *an extension of* χ *to an irreducible character of* $AGL_n(k_2)$. *Then, there exists a unique irreducible character* ψ_{χ} *of* $U_n(k_2)$ *which depends only on* χ *and satisfies*

$$\tilde{\chi}(\sigma x)=\varepsilon(\tilde{\chi})\psi_{\chi}(n(x)) \qquad (x\in GL_n(k_2)),$$

where $\varepsilon(\tilde{\chi})=\pm 1$ *and* $n(x)$ *is an arbitrary element of* $U_n(k_2)$ *conjugate to* $x^{\sigma}x$ *in* $GL_n(k_2)$. *Moreover, the correspondence* $\chi\rightarrow\psi_{\chi}$ *is a bijection between the set of* σ*-fixed irreducible characters of* $GL_n(k_2)$ *and the set of irreducible characters of* $U_n(k_2)$.

This paper consists of five sections. § 1 is a recollection of some known

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