J. Math. Soc. Japan
Vol. 29, No. 2, 1977

A note on Ξ-product

By Syōji SAN-OU

(Received Feb. 5, 1976)
(Revised May 25, 1976)

§ 1. Introduction.

Let $\{X_\alpha\}_{\alpha\in A}$ be a family of topological spaces. We denote the box product space by $B_\alpha X_\alpha$. For $p\in B_\alpha X_\alpha$ let Ξ_p be the subspace $\{x\in B_\alpha X_\alpha : x_\alpha\neq p_\alpha$ for at most finitely many $\alpha\}$ of $B_\alpha X_\alpha$.

Recently E. K. van Douwen [4] showed Ξ_p is stratifiable if each X_α is a metrizable space and p is any point of $B_\alpha X_\alpha$.

In this paper we shall show the followings.

(A) *Let* $\{X_\alpha\}_{\alpha\in A}$ *be a family of metrizable spaces and* p *be any point of* $B_\alpha X_\alpha$. *Then* Ξ_p *is an* M_1-*space* (Corollary 3.3).

(B) *Let* $\{X_\alpha\}_{\alpha\in A}$ *be a family of* M_2-*spaces and* p *be any point of* $B_\alpha X_\alpha$. *Then* Ξ_p *is an* M_2-*space* (Corollary 3.4).

(C) *There exists an* M_1-*space* X *and a closed subset* A *of* X *such that* (X, A) *is not semi-canonical* (Example 4.1).

Both (A) and (B) strengthen the theorem of E. K. van Douwen [4]. (C) shows that the Lemma of M. R. Cauty [3] is false.

Throughout of this paper all topological space are $\boldsymbol{T}_1$ and N denote the natural numbers.

The auther thanks the referee for pointing out that (B) follows from our original version.

§ 2. Preliminaries.

The symbol $\Pi_\alpha X_\alpha$ denotes the set product of the family of the sets $\{X_\alpha\}_{\alpha\in A}$.

LEMMA 2.1. *Let* $\{X_\alpha\}_{\alpha\in A}$ *be a family of topological spaces and let* $\mathfrak{P}_\alpha$, $\alpha\in A$, *be a cushioned collection* (see [3]) *consisting of ordered pairs of subsets* $P_\alpha=(P_\alpha^1, P_\alpha^2)$ *of* X_α. *Then for any subspace* Y *of* $B_\alpha X_\alpha$ $\{(\Pi_\alpha P_\alpha^1\cap Y, \Pi_\alpha P_\alpha^2\cap Y) : P=(P_\alpha^1, P_\alpha^2)\in\Pi_\alpha\mathfrak{P}_\alpha\}$ *is a cushioned collection of* Y.

PROOF. For a given arbitrary subfamily $\mathfrak{P}$ of $\Pi_\alpha\mathfrak{P}_\alpha$, we have to show that

$$cl_Y(\cup\{\Pi_\alpha P_\alpha^1\cap Y : P\in\mathfrak{P}\})\subset\cup\{\Pi_\alpha P_\alpha^2\cap Y : P\in\mathfrak{P}\}\,.$$

Let p be any point of $Y-\cup\{\Pi_\alpha P_\alpha^2\cap Y : P\in\mathfrak{P}\}$. For any $\alpha\in A$, let