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Embeddings of infinite-dimensional manifold pairs and remarks on stability and deficiency

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Abstract. In this paper, we treat of an E-manifold pair (M, N) with N a Z-set in M where E is an infinite-dimensional locally convex linear metric space which is homeomorphic to E^{ω} or E_f^{ω}. And we study the condition under which M can be embedded in E such that N is the topological boundary under the embedding (Anderson's Problem in [2]). Moreover we extend the results on topological stability and deficiency, the Homeomorphism Extension Theorem and the results in [18].

§ 0. Introduction.

For each space X, we denote by X^{ω} the countable infinite product of X by itself. And for each space X with a base point 0, $X_f^{\omega}=\{(x_i)\in X^{\omega} \mid x_i=0$ for almost all $i\}$. A closed subset K of a space X is a *Z-set in X* if for each non-empty homotopically trivial open set U, $U\backslash K$ is also non-empty and homotopically trivial ([1]). An *E-manifold* is a paracompact manifold modelled on a space E. As a modelled space, let E be a locally convex linear metric space (LCLMS) homeomorphic ($\cong$) to E^{ω} or E_f^{ω}. In an E-manifold pair (M, N), N is a Z-set in M if and only if N is a collared closed set in M (collared in the sense of M. Brown [7] (see 4-4 in this paper). Then (M, N) may be considered as a manifold-with-boundary, N being the boundary. Thus the study of E-manifolds-with-boundary becomes the study of such E-manifold pairs. However circumstances of infinite-dimensional manifolds are different from finite-dimensional case (e.g., see Examples 1 and 2 of Sect. 7 in this paper).

In this paper, we study the problem for such an E-manifold pair (M, N): *Under what condition can M be embedded in E such that N is a topological boundary under the embedding*? This problem for separable l^2-manifold pairs was raised by R. D. Anderson in [2]. In the previous paper [20], we found a sufficient condition of this problem: *N contains some deformation retract of M.* And we saw that even if N is homeomorphic to E, M cannot always be embedded such a way. But we have an easy example of E-manifold pairs which do not satisfy the above condition but which can be embedded such a way.