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Nonlinear perturbations of linear evolution systems*

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It is the purpose of this paper to study the existence and behavior of "mild" solutions to abstract differential equations in Banach spaces. Roughly speaking, the "right hand side" of these equations are assumed to be the sum of two operators, one being the generator of a linear (or affine) evolution system and the other being continuous and nonlinear. The results and techniques established here are improvements of those given by the author in [2].

In §1 we indicate the basic notations, and the fundamental results on the existence of approximate solutions are developed in §2. Criteria for local existence of solutions is given in §3, and §4 is devoted to establishing a relationship of these ideas to inequalities. An example illustrating the abstract results is studied in §5.

§1. Preliminaries.

Let X be a Banach space over the real or complex field and let $|\cdot|$ denote the norm on X. Also, let $-\infty<a<b\leqq+\infty$ and let $\Delta=\{(t, s): a\leqq s\leqq t<b\}$. Throughout this paper $T=\{T(t, s):(t, s)\in\Delta\}$ is a family of bounded linear operators on X which satisfy

(T1) $T(t, t)=I$ and $T(t, s)T(s, r)=T(t, r)$ for $a\leqq r\leqq s\leqq t<b$;

and

(T2) the map $(t, s)\rightarrow T(t, s)x$ is continuous from Δ into X for each $x\in X$.

From (T2) and the principle of uniform boundedness we also have that

(T3) for each $c\in[a, b)$ there is a number $M_c\geqq 1$ such that

$$\|T(t, s)\|\equiv\sup\{|T(t, s)x|:|x|\leqq 1\}\leqq M_c \text{ for all } a\leqq s\leqq t\leqq c.$$

Now suppose that $\alpha:[a, b)\rightarrow X$ is continuously differentiable, $\beta:[a, b)\rightarrow X$ is continuous, and define the family $S=\{S(t, s):(t, s)\in\Delta\}$ of (affine) mappings on X by

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