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Unbounded representations of symmetric $*$-algebras

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§ 1. Introduction.

In [**16**], R. T. Powers began studying a representation of a $*$-algebra as an algebra of unbounded operators on a Hilbert space. A class of symmetric unbounded operator algebras (called symmetric $\sharp$-algebras, $EC^{\#}$-algebras, $EW^{\#}$-algebras, EC^{*}-algebras and EW^{*}-algebras) have been studied by P. G. Dixon [**3, 4**], the author [**9, 10, 11, 12**] and others.

In this paper we shall study unbounded representations of symmetric $*$-algebras. Let A be a symmetric $*$-algebra and let π be a representation of A on a Hilbert space $\mathfrak{H}$. Then we divide π into the following three types. If $\pi(x)$ is a bounded operator for all $x\in A$, then π is called a bounded representation. If π is unitarily equivalent to the direct sum of bounded representations of A, then π is called a weakly unbounded representation of A. If π has not any bounded subrepresentation of π, then π is called a strictly unbounded representaton. In § 3 we obtain the following theorems.

THEOREM 3.11. *If π is closed, then it is unitarily equivalent to the direct sum of strongly cyclic closed representations.*

THEOREM 3.13. *If π is closed, then there are a weakly unbounded closed representation π_1 of A and a strictly unbounded closed representation π_2 of A such that π is unitarily equivalent to the direct sum of π_1 and π_2.*

In § 4, we shall consider the relation of positive linear functionals and representations. Let f be a positive linear functional on A. By Gelfand-Segal construction there is a strongly cyclic closed representation π_f of A on a Hilbert space $\mathfrak{H}_f$ with a strongly cyclic vector ξ_f such that $f(x)=(\pi_f(x)\xi_f|\xi_f)$ for all $x\in A$. We divide f into the following three types. An f is said to be relatively bounded if π_f is bounded. An f is said to be approximately relatively bounded if an f is contained in the weak closure of $\{g; f\geqq g\geqq 0$ and g is relatively bounded$\}$. An f is said to be strictly relatively unbounded if there is not any non-zero positive linear functional g such that $f\geqq g$ and g is relatively bounded. The primary purpose of this section is to show the following two theorems.

THEOREM 4.4. *There exists a decomposition of f such that $f=f_1+f_2$, f_1 is*