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On irreducible unitary characters of a certain group extension of $GL(2, \boldsymbol{C})$

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Introduction.

0-1. Let $G=GL(2, \boldsymbol{C})$ be the complex general linear group of order 2. Denote by $\langle\sigma\rangle$ a group of automorphisms of G generated by the complex conjugation σ. Let $G^{\sim}$ be the semi-direct product of G with $\langle\sigma\rangle$. More precisely, $G^{\sim}$ is the group whose underlying set is $G\times\langle\sigma\rangle$ and whose composition law is given by $(g, \tau)(g', \tau')=(g^{\tau'}g', \tau\tau')$. Then $G^{\sim}$ is a disconnected Lie group which has G as a connected component of the identity element. Let T be an irreducible unitary representation of $G^{\sim}$. Then the restriction of T to G is either an irreducible representation of G or the direct sum of two mutually inequivalent irreducible representations of G. Accordingly, T is said to be *of the first* or *the second kind*. In the following, we assume T to be of the first kind. For each smooth and compactly supported function f on G, it is known that the operator $\int_G f(g)T(g, \sigma)dg$ is a trace operator acting on the representation space of T (dg is an invariant measure on G). Moreover it is shown that there exists a locally integrable function trace $T(g, \sigma)$ on G such that

$$\operatorname{trace}\int_G f(g)T(g, \sigma)dg=\int_G f(g)\operatorname{trace} T(g, \sigma)dg\,.$$

On the other hand, set $G_R=GL(2, \boldsymbol{R})$. It is known that, for any irreducible unitary representation r of G_R, there exists a locally summable class function trace $r(x)$ on G_R such that

$$\operatorname{trace}\int_{G_R}\varphi(x)r(x)dx=\int_{G_R}\varphi(x)\operatorname{trace} r(x)dx$$

for any smooth and compactly supported function φ on G_R (dx is an invariant measure on G_R). We extend a class function trace r on G_R to a class function on G_C by setting

$$\operatorname{trace} r(g)=\begin{cases}\operatorname{trace} r(x) & \text{if } g \text{ is conjugate to } x\in G_R \text{ in } G_C\,,\\ 0 & \text{otherwise.}\end{cases}$$