J. Math. Soc. Japan
Vol. 29, No. 1, 1977

On a construction of a recurrent potential kernel by mean of time change and killing

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(Received Feb. 14, 1976)

§1. Introduction.

Let E be a locally compact Hausdorff space with countable base, $\mathcal{E}$ be the σ-field of Borel subsets of E and $X=(\Omega, \mathcal{F}, \mathcal{F}_t)_{t\geqq 0}, (X_t)_{t\geqq 0}, (\theta_t)_{t\geqq 0}, (P^x)_{x\in E})$ be a Hunt process on $(E, \mathcal{E})$. The constructions of the (weak) potential kernel of X were given by many authors ([6], [9], [11], [13]). In this paper we shall give a construction by mean of time change and killing. Let $A=(A_t)_{t\geqq 0}$ be a non-trivial non-negative continuous additive functional of X such that $A_t<\infty$ a.s. for all $t<\infty$. Let $K_{P,C}^0$ and $G_{P,C}^0$ be the resolvent of the time changed process corresponding to the additive functional A^C and the potential kernel of the subprocess of X corresponding to the multiplicative functional $(e^{-PA_t^C})_{t\geqq 0}$, respectively, where A^C is defined by

$$A_t^C=\int_0^t I_C(X_s)dA_s$$

for a Borel subset C of E. Then for a suitably chosen Borel subset C of E there exists a potential kernel K_C of $K_{1,C}^0$ restricted to $C\times C$ and the kernel defined by

$$K(x, dy)=G_{1,C}^0(x, dy)+K_{1,C}^0K_CG_{1,C}^0(x, dy)$$

is a potential kernel of X. If there exists a dual Hunt process $\hat{X}$ of X relative to the invariant measure μ of X then the kernels K and $\hat{K}$ defined as above by A^C and $\hat{A}^C$ are in dual relative to μ, where $\hat{A}$ is the dual continuous additive functional of A. By these method, we can construct, explicitly, the potential kernel of one dimensional non-singular diffusion processes.

§2. Construction of a potential kernel.

Throughout in this paper we shall assume that X is a recurrent Hunt process on $(E, \mathcal{E})$, that is, it satisfies the following equivalent conditions (Azema-Duflo-Revuz [1], Blumenthal-Getoor [5] problems II.4.17-4.20).