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On subdiagonal algebras associated with flows in operator algebras

By Shinzô KAWAMURA and Jun TOMIYAMA

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Abstract. The noncommutative Hardy spaces $H^{\infty}(\alpha)$ and $H^{1}(\alpha)$ are introduced with respect to a σ-weakly continuous flow $\alpha=\{\alpha_t\}$ of $*$-automorphisms of a von Neumann algebra. In case that the algebra is α-finite the algebra $H^{\infty}(\alpha)$ becomes a maximal subdiagonal algebra. The concept of C^*-subdiagonal algebras will also be given for C^*-algebras as a noncommutative counterpart of the algebras of generalized analytic functions. Examples of maximal C^*-subdiagonal algebras and their structures are discussed.

§ 1. Introduction.

Let B (resp. M) be a C^*-algebra (resp. a von Neumann algebra) and $\{\alpha_t\}$ be a flow by which we mean a strongly continuous (resp. σ-weakly continuous) one parameter group of $*$-automorphisms of the algebra. Let $A(\alpha)$ be the set of all elements of B with nonnegative spectrum with respect to $\{\alpha_t\}$, then it turns out to be a closed subalgebra of B such that the intersection $A(\alpha)\cap A(\alpha)^*$ is the C^*-subalgebra of fixed elements of α_t. The algebra has been studied by several authors, especially in the case where B is a commutative C^*-algebra as a function algebra with an analyticity ([8], [17], [18], [4] etc). A prototype of such algebra is the classical disk algebra on the unit circle, or the algebra of generalized analytic functions, which is determined by the rotation flow on the unit circle (periodic), or by an almost periodic flow on the compact dual of an ordered discrete group. Furthermore, in these cases, the Hardy spaces H^{∞}'s are also considered as such algebras of elements with nonnegative spectrums with respect to the weak $*$ continuous flows of the corresponding L^{∞}-spaces, i.e. of commutative von Neumann algebras. Now, a counterpart of the H^{∞} algebra or more generally of a weak $*$ Dirichlet algebra has been studied by Arveson [2] in the context of von Neumann algebras as a theory of subdiagonal algebras, in which, however, the above rôle of flows has not been discussed. A subdiagonal algebra is a pair $(\mathfrak{A}, \varepsilon)$ where $\mathfrak{A}$ is a subalgebra of a von Neumann algebra M such that $\mathfrak{A}+\mathfrak{A}^*$ is σ-weakly dense in M, and where ε is a homomorphism of $\mathfrak{A}$ into $\mathfrak{A}\cap\mathfrak{A}^*$ which extends in a suitable way to M.