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On the intransitive Lie algebras whose transitive parts are infinite and primitive

By Tohru MORIMOTO

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Introduction.

At the beginning of this century E. Cartan developed the theory of infinite Lie groups in his series of papers [1], [2], [3], and one of his main achievements was the classification of the simple infinite Lie groups [3].

With the increasing interest in this field, modern formulations and treatments were made around 1960 (especially in [9], [5], [13]), and then the rigorous and systematic proof of the classification of the simple infinite Lie groups was given in the transitive case ([8], [6], [10]).

On the other hand in the intransitive case, only a few attempts were made by several authors. N. Tanaka [14] and K. Ueno [15] studied the generalized G-structures towards the equivalence problem of the intransitive Lie groups. V. Guillemin [7] studied a Jordan-Hölder decomposition of the transitive Lie algebras and introduced simple intransitive Lie algebras occuring in the decomposition. But in his treatment the intransitive Lie algebras considered are limited to those which are ideals of some Lie algebras.

It seems that there are no satisfactory formulations and treatments of the intransitive infinite Lie groups. In particular, in spite of the early work of Cartan, the classification of the simple intransitive infinite Lie groups has not yet been rigorously settled.

The purpose of this paper is to contribute to the classification of the simple intransitive infinite Lie groups.

According to Cartan [3], the classification problem is divided into the following three problems:

(I) To reduce the problem to (II) and (III).

(II) To determine all intransitive Lie groups whose restrictions to the orbits are infinite and primitive.

(III) To determine all intransitive infinite Lie groups whose restrictions to the orbits are finite, simple, and simply transitive on the orbits.

In this paper we take up the problem (II). We shall formulate it in the category of the formal Lie algebras, and carry it through.