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On the finite element method for parabolic equations, I; approximation of holomorphic semi-groups[1)]

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§ 1. Introduction.

In the present paper and a few papers to follow, we shall make an operator theoretical study of the finite element method applied to the initial boundary value problems for partial differential equations of parabolic type.

The particular objectives of the present paper are twofold. Firstly, we intend to develop a method of error analysis of a general nature which is in conformity with the theory of holomorphic semigroups (cf. Yosida [21] or Kato [13]). Specific descriptions of this method would be only possible after introduction of necessary notions and notation as in the following sections. However, it seems to be appropriate to give a few comments here of the motivation or the idea of our method. The initial boundary value problem which we are going to consider can be formulated as an evolution equation in a Hilbert space X of the following form with an m-sectorial operator A (Kato [13], Chap. V, VI):

$$\begin{cases} \dfrac{du}{dt}+Au=0\,, & (t>0)\,, \\ u(0)=a\,, \end{cases}$$

where t is the time variable and a is the initial value. The solution $u:[0, \infty)\rightarrow X$ is given in terms of the semigroup e^{-tA} generated by $-A$ as

$$u(t)=e^{-tA}a\,.$$

Reflecting the parabolicity of the original equation, this semigroup e^{-tA} is a holomorphic semigroup and it admits of the integral representation (the Dunford integral)

$$e^{-tA}=\frac{1}{2\pi i}\int_{\Gamma}e^{-tz}(z-A)^{-1}dz \qquad (t>0)\,.$$

1) A part of this paper was reported by the first author at the IRIA symposium in December, 1975, 2nd international symposium on computing methods in applied sciences and engineering (cf. Fujita [8]).