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Cohomology of finitely generated Kleinian groups with an invariant component

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Preliminaries. Let G be a non-elementary finitely generated Kleinian group with the region of discontinuity $\Omega(G)$ and let $B_q(\Omega(G), G)$ be the space of bounded holomorphic automorphic forms of weight $-2q$ for G operating on $\Omega(G)$, where $q(\geqq 2)$ is an integer. We denote by Π_{2q-2} the vector space of complex polynomials in one variable of degree at most $2q-2$. Clearly Π_{2q-2} is a G-module with $(v\cdot\gamma)(z)=v(\gamma(z))\gamma'(z)^{1-q}$ for $v\in\Pi_{2q-2}$ and $\gamma\in G$.

Now we can form the (first) cohomology space $H^1(G, \Pi_{2q-2})$, that is, $H^1(G, \Pi_{2q-2})$ is the space of cocycles $Z^1(G, \Pi_{2q-2})$ factored by the space of coboundaries $B^1(G, \Pi_{2q-2})$. Let p be an element of $Z^1(G, \Pi_{2q-2})$. If p satisfies the condition $p|_{G_0}\in B^1(G_0, \Pi_{2q-2})$ for any parabolic cyclic subgroup G_0 of G, then we say that p belongs to $PZ^1(G, \Pi_{2q-2})$, the space of parabolic cocycles. We denote by $PH^1(G, \Pi_{2q-2})$, the space of parabolic cohomology, that is, the space of parabolic cocycles factored by the space of coboundaries. From this definition, we see

$$\dim PH^1(G, \Pi_{2q-2})=\dim PZ^1(G, \Pi_{2q-2})-\dim B^1(G, \Pi_{2q-2}).$$

Further, for a non-elementary Kleinian group G, the equality

$$\dim B^1(G, \Pi_{2q-2})=2q-1$$

is known (see Bers [**1**]).

We have the so-called Bers' map

$$\beta^*: B_q(\Omega(G), G)\longrightarrow PH^1(G, \Pi_{2q-2})$$

which is anti-linear and injective (see Bers [**1**] and Kra [**2**]).

Throughout this paper, we call the group consisting only of the identity to be trivial. This group is, of course, a cyclic group. Let H be a cyclic subgroup of a Kleinian group G. The interior B of a closed topological disc is called a precisely invariant disc under H if $h(\bar{B}-\Lambda(H))=\bar{B}-\Lambda(H)$ for $h\in H$ and $g(\bar{B}-\Lambda(H))\cap(\bar{B}-\Lambda(H))=\emptyset$ for $g\in G-H$, where $\bar{B}$ is the closure of B, $\Lambda(H)$ is the limit set of H and $\bar{B}-\Lambda(H)\subset\Omega(G)$.

The following Maskit's Combination Theorems play a fundamental role in