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On locally symmetric Kaehler submanifolds in a complex projective space

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We denote by $M_n(c)$ an n-dimensional Kaehler manifold of constant holomorphic sectional curvature c, which is called a *complex space form.* An isometric and holomorphic immersion of a Kaehler manifold into a Kaehler manifold is said to be a *Kaehler immersion.* The study of Kaehler submanifolds immersed into a complex space form arose from a work of E. Calabi [**5**], who proved the local rigidity theorem to the effect that a Kaehler submanifold with analytic metric imbedded into $M_N(c)$ is locally rigid, and found the necessary and sufficient condition for a simply connected Kaehler manifold to be globally imbedded into a complete and simply connected complex space form as a Kaehler submanifold. Moreover, he completely classified Kaehler imbeddings of an n-dimensional complex projective space P_n into an N-dimensional complex projective space P_N.

After a while, B. Smyth [**23**] determined all complete and simply connected Einstein Kaehler hypersurfaces immersed into a complete and simply connected complex space form from the differential geometric point of view. The corresponding local theorem was proved by S. S. Chern [**8**]. As for extensions of these theorems, there are results of K. Nomizu and B. Smyth [**20**] and T. Takahashi [**24**]. With relation to these works, Kaehler submanifolds immersed in a complex space form are studied from various standpoints. In particular, K. Ogiue investigated these topics systematically, and related results are collected in [**22**]. Furthermore, concerning Einstein Kaehler submanifolds in P_N, J. Hano [**13**] obtained an interesting and suggestive result, and the first named author and K. Ogiue [**18**] studied the local version of Calabi's classification mentioned above. We note here that all examples of Einstein Kaehler submanifolds in P_N we know so far are symmetric.

Now, a complex projective space is one of the simplest examples of compact irreducible Hermitian symmetric spaces. Moreover, it is known that they

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