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The Levi problem for the product space of a Stein space and a compact Riemann surface

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Introduction.

Since Oka [9] solved the Levi problem for unramified domains over $\boldsymbol{C}^n$, many mathematicians extended Oka's theorem (cf. Andreotti-Narasimhan [1], Narasimhan [8]). On the other hand, recently Nakano [7] obtained the vanishing theorems for weakly 1-complete manifolds. The aim of the present paper is to give a solution of the following Levi problem for the product space of a Stein space and a compact Riemann surface.

THEOREM. *Let S be a Stein space, R be a compact Riemann surface and X be the product space of S and R. $\pi_1: X\rightarrow S$ denotes the projection of X onto S. Let D be a domain of X. Then the following assertions* (1), (2) *and* (3) *are equivalent:*

(1) *D is weakly* 1*-complete.*

(2) *D is holomorphically convex.*

(3) *Either D is a Stein space or $D=\pi_1(D)\times R$, $\pi_1(D)$ being a Stein space.*

This theorem is a generalization of the previous paper [12] and the result of Matsugu [6].

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§ 1. The Levi problem for relatively compact domains on a weakly 1-complete space.

All complex analytic spaces considered in this paper are supposed countable at infinity.

DEFINITION [7]. Let X be a complex analytic space and ϕ be a C^∞ function on X. We say that X is complete with the exhausting function ϕ if and only if

$$X_c:=\{x\in X;\ \phi(x)<c\}$$

is relatively compact for every $c\in\boldsymbol{R}$. Moreover if ϕ is plurisubharmonic on