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Extremizations and Dirichlet integrals on Riemann surfaces

Dedicated to Professor Leo Sario on his 60th birthday

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Consider a subregion W of a Riemann surface R with an analytic relative boundary ∂W, compact or noncompact. We denote by $H(W;\partial W)$ the class of continuous functions v on R harmonic on W and vanishing on $R-W$. The operator μ from a domain in $H(W;\partial W)$ into $H(R)$ given by

(1) $$\mu v=\lim_{\Omega\to R}H_{v}^{\Omega}$$

is referred to as the *extremization* relative to (R, W) in the Kuramochi terminology, where $\{\Omega\}$ is the directed net of regular subregions of R and H_{v}^{Ω} is the harmonic function on Ω with boundary values v on $\partial\Omega$. Let $HX(W;\partial W)$ be the subclass of $H(W;\partial W)$ consisting of members with the property $X=D$ or BD where D means the finiteness of the *Dirichlet integral*

(2) $$D_{R}(v)=\int_{R}dv\wedge{}^{*}dv\ ,$$

B the boundedness, and BD both B and D. As a consequence of the Dirichlet principle the domain of μ contains $HX(W;\partial W)$ and the range of $\mu_{X}=\mu|HX(W;\partial W)$ is contained in $HX(R)$ ($X=D$ and BD), i. e.

(3) $$\mu_{D}: HD(W;\partial W)\longrightarrow HD(R)$$

and also

(4) $$\mu_{BD}: HBD(W;\partial W)\longrightarrow HBD(R)$$

are linear operators, which are *injective*, positive, and isometric with respect to the supremum norm on R; and the former is an extension of the latter. Here we recall the following theorem of Royden: the classes $HBD(W;\partial W)$ and $HBD(R)$ are *dense* in $HD(W;\partial W)$ and $HD(R)$, respectively, with respect to the Dirichlet seminorm $D_{R}(\cdot)^{1/2}$ and the supremum seminorms $\sup_{K}|\cdot|$ for all compact subsets K of R. In view of this we naturally come up with the following

QUESTION. *Does the surjectiveness of μ_{BD} imply that of μ_{D}?*