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C^{∞}-approximation of continuous ovals of constant width

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§ 1. Introduction.

Let M be an oval (i.e., a closed convex curve) in a Euclidean 2-space E^2. For a point x of M a straight line l passing through x is called a supporting line at x if M is contained in one of the half planes determined by l. If M is a C^1-curve, then tangent lines are supporting lines. M is said to have constant width, if the distance between each pair of parallel supporting lines is constant. Examples of continuous ovals of constant width are Reuleaux triangles, Sallee constructions (cf. [7], and also B.B. Peterson [4]), and so on.

We prove C^{∞}-approximation theorem:

THEOREM A. *Let M be a continuous oval of constant width H in E^2. Then, for any positive number δ, we can construct a C^{∞}-oval $M^{\#}$ of constant width H in the δ-neighborhood of M in E^2.*

THEOREM B. *In Theorem A, if M is symmetric with respect to a straight line m in E^2, then $M^{\#}$ can be constructed so that $M^{\#}$ is symmetric with respect to m.*

A generalization of an oval of constant width to higher dimension is a hypersurface of constant width in a Euclidean n-space E^n. If M is a continuous oval of constant width in $E^2 \subset E^n$, which is symmetric with respect to the x^1-axis, then one gets a continuous hypersurface of constant width in E^n as its revolution hypersurface with respect to the x^1-axis in E^n.

By Theorem B we obtain

THEOREM C. *If a continuous hypersurface M of constant width H is a revolution hypersurface in E^n, then for any positive number δ, we can construct a revolution C^{∞}-hypersurface $M^{\#}$ of constant width H in the δ-neighborhood of M in E^n.*

In the last section we mension about twin hypersurfaces which are generalizations of hypersurface of constant width.