J. Math. Soc. Japan
Vol. 28, No. 2, 1976

A test of Picard principle for rotation free densities, II

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(Received April 24, 1975)

A nonnegative locally Hölder continuous function $P(z)$ on the punctured closed unit disk $0<|z|\leqq 1$ will be referred to as a *density* on $0<|z|\leqq 1$. The dimension of the half module of nonnegative solutions u of $\Delta u(z)=P(z)u(z)$ on $0<|z|<1$ with vanishing boundary values zero on $|z|=1$ is called the *elliptic dimension* of P at $z=0$, $\dim P$ in notation. After Bouligand we say that the *Picard principle* is valid for P if $\dim P=1$. For *rotation free* densities $P(z)$, i. e. densities $P(z)$ satisfying $P(z)=P(|z|)$ on $0<|z|\leqq 1$, it was shown in [**20**] that

$$\dim P=1+\alpha(P)\cdot \mathfrak{c} \tag{1}$$

where $\mathfrak{c}$ is the cardinal number of continuum and $\alpha(P)$ is the quantity in $[0, 1)$ associated with P which is referred to as the *singularity index* of P. In particular the Picard principle is valid for rotation free densities P if and only if $\alpha(P)=0$. In this context it is important to provide practical tests for $\alpha(P)=0$ and also for $\alpha(P)>0$. The purpose of this paper is, as a continuation of the paper [**22**] with the same title, to contribute to this latter subject.

There exists a unique bounded solution $e_P(z)$, referred to as the *P-unit*, of $\Delta u=Pu$ on $0<|z|<1$ with boundary values 1 on $|z|=1$. The first of our main results in this paper is the following complete characterization of $\alpha(P)=0$ in terms of e_P given in § 2: The Picard principle is valid for a rotation free density $P(z)$ if and only if

$$\int_0^1 \frac{dr}{r\left(r\frac{d}{dr}\log e_P(r)+1\right)}=\infty . \tag{2}$$

As an application of this we can settle the validity of the *order comparison theorem* in the affirmative for rotation free densities (cf. [**20**], [**21**], [**22**]): If $P_1(z)$ and $P_2(z)$ are rotation free densities on $0<|z|\leqq 1$ such that

$$c^{-1}P_1(z)\leqq P_2(z)\leqq cP_1(z)$$

*) The work was done while this author was a Research Fellow at Nagoya University in 1974 supported by Japan Ministry of Education.