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On the prolongation of local holomorphic solutions of partial differential equations, II, prolongation across the pluri-harmonic hypersurface

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§1. Introduction.

Recently many results about the holomorphic continuation of solutions of a partial differential equation in a complex domain are obtained. Let Ω be a domain in C^n with a smooth boundary, denoted by $\partial\Omega$ and $P(z, D)$ be a linear partial differential operator whose coefficients are holomorphic in some neighborhood of a given point $p\in\partial\Omega$. The main results already obtained are the following:

(i) if the boundary $\partial\Omega$ is *non-characteristic* with respect to $P(z, D)$ at p, then every holomorphic solution $u(z)$ of $P(z, D)u(z)=0$ in Ω is holomorphic at p. (Zerner [5].)

(ii) if the boundary $\partial\Omega$ is *simply characteristic* at p and the normal curvature in some direction in the complex bi-characteristic curve is negative, then every solution $u(z)$ of $P(z, D)u(z)=0$ in Ω is holomorphic at p. (Tsuno [3].)

(iii) if the boundary $\partial\Omega$ is *strictly pseudo-convex* and *simply characteristic* at p and the normal curvature in every direction in the complex bicharacteristic curve is positive, then under some additional conditions we can construct a solution $u(z)$ of $P(z, D)u(z)=0$ in Ω which is not holomorphic at p. (Tsuno [3].)

(iv) if the boundary $\partial\Omega$ is *real-analytic*, P. Pallu de La Barrière ([1] and his thesis) studied the existence and prolongation of holomorphic solutions in the framework of the hyperfunction theory and obtained the following result as an application (Théorème 4.1 in his thesis): under the same situation of our theorem in the next section, if the holomorphic function $P_m(z, \mathrm{grad}_z\Phi(z))$ has at most *simple zero* on the hypersurface $\{\Phi(z)=0\}$, then every solution $u(z)$ of $P(z, D)u(z)=0$ in Ω is holomorphic at 0.

In this paper we are concerned with the case where the boundary $\partial\Omega$ is