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On the abstract linear evolution equations in Banach spaces

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§ 0. Introduction.

The objective of the present paper is to construct the evolution operator associated with an evolution equation

(E) $$du/dt+A(t)u=f(t)$$

in a Banach space X. Here $u=u(t)$ and $f(t)$ are functions on $[0, T]$ to X and $A(t)$ is a function on $[0, T]$ to the set of linear operators acting in X. We assume (E) is of parabolic type, that is, $-A(t)$ are all infinitesimal generators of analytic semi-groups of bounded linear operators on X.

This problem has been considered already in many papers, for instance, [1], [2] and [3]. The main assumption of [1] is that the inequality

$$\|A(t)^{\rho}dA(t)^{-1}/dt\|\leqq N \tag{0.1}$$

is valid with some constant $\rho\in(0,1]$; and those of [2] are that the inequality of the form

$$\begin{aligned}&\|A(t)(\lambda-A(t))^{-1}(dA(t)^{-1}/dt)A(t)(\lambda-A(t))^{-1}\|\\&=\|\partial/\partial t(\lambda-A(t))^{-1}\|\leqq N/|\lambda|^{\rho}\end{aligned} \tag{0.2}$$

is valid with some constant $\rho\in(0,1]$ and $dA(t)^{-1}/dt$ is Hölder continuous. In [3] the following conditions are assumed: the domain of $A(t)^{\rho}$ is independent of t for some $\rho=1/m$ where m is a positive integer, and $A(t)^{\rho}A(0)^{-\rho}$ is Hölder continuous in t.

In this paper we assume the inequality

$$\|A(t)(\lambda-A(t))^{-1}dA(t)^{-1}/dt\|\leqq N/|\lambda|^{\rho} \tag{0.3}$$

with a constant $\rho\in(0,1]$. This inequality (0.3) is slightly weaker than the inequality (0.1), for (0.1) implies (0.3) by the equation

$$A(t)(\lambda-A(t))^{-1}dA(t)^{-1}/dt=A(t)^{1-\rho}(\lambda-A(t))^{-1}A(t)^{\rho}dA(t)^{-1}/dt$$

and the estimation

$$\|A(t)^{1-\rho}(\lambda-A(t))^{-1}\|\leqq M/|\lambda|^{\rho}.$$