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On a theorem of Alekseevskii concerning conformal transformations

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The purpose of this note is to give a direct proof to a theorem of Alekseevskii which asserts existence of a special kind of neighborhoods around a certain point of a Riemannian manifold.

Let M be a Riemannian manifold of dimension $m\geqq 3$. The following is known as Lichnerowicz's conjecture: *If the largest connected group $C_0(M)$ of conformal transformations of M is essential* (See §1 for the meaning of terminology), *then M is conformal either to a Euclidian sphere $\boldsymbol{S}^m$ or to a Euclidian space $\boldsymbol{E}^m$.* This conjecture is affirmatively answered by Lelong-Ferrand [2] and by Obata [3] in the case when M is compact, and also by others under some additional conditions (cf. [4]). Recently Alekseevskii [1] tried to assure the conjecture for the most general case. The theorem we shall establish in this paper is stated in a slightly weaker form in the paper [1] with a proof which seems incomplete. We shall show, in a way slightly different from Alekseevskii's, how our theorem is applied to a proof of Lichnerowicz's conjecture for the general case under the assumption that M admits an essential one-parameter subgroup of $C_0(M)$.

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§1. Statement of Theorem.

Let M be a Riemannian manifold. Throughout this paper, manifolds, functions etc. are assumed to be of class C^∞. We shall denote by $C(M)$ the group of all conformal transformations of M endowed with the compact-open topology and $C_0(M)$ its connected component of the identity. A subgroup G (resp. an element ϕ) of $C(M)$ is said to be *essential*, if G (resp. ϕ) is not contained in the group of all isometric transformations of the manifold M endowed with any Riemannian metric conformal to the original one.

Let Ψ be a family of diffeomorphisms of a Riemannian manifold M which leave a point $p\in M$ fixed. A neighborhood U of p is said to be *Ψ-admissible*