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A class of infinitesimal generators of one-dimensional Markov processes

By Heinz LANGER

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In this note we consider operators $\mathfrak{A}$ of the form

$$(\mathfrak{A}f)(x)=(D_mD_xf)(x)+b(x)(D_xf)(x)+\\+\int_0^1(f(y)-f(x)-(y-x)(D_xf)(x))\frac{n_x(dy)}{\varphi_x(y)},\qquad x\in[0,1] \qquad (1)$$

in spaces of continuous functions over the interval $[0, 1]$ (for the properties of m, b, n_x and the definition of φ_x see the beginning of 2.). It is shown, that $\mathfrak{A}$ restricted by two boundary conditions

$$\Phi_0(f)=0, \qquad \Phi_1(f)=0 \qquad (2)$$

of Feller-Ventcel-type (see (13)) is the infinitesimal generator of a strongly continuous nonnegative contraction (s. c. n. c.) semigroup in the subspace of $C_{[0,1]}$, which is defined by the boundary conditions (2).

Similar results (in cases without boundary conditions) can be found in [**1**]. As in [**1**] (or [**2**]) we use a perturbation type argument, but here it does not consist in a "smallness" condition on the perturbing operator B (with respect to the unperturbed operator A), but in the compactness of the operator $B(\lambda I-A)^{-1}$ ($\lambda>0$) (see theorem 1 below).

To avoid technical complications, we consider only the case of a strongly increasing and continuous function m in (1). The general case of arbitrary nondecreasing m can be treated similarly (comp. [**1**])*).

1. In this section we consider a Banach space $\mathfrak{B}$ with a certain fixed semi-inner product $[f, g]$, $f, g\in\mathfrak{B}$ ([**3**], IX. 8). An operator A in $\mathfrak{B}$ is called dissipative (with respect to $[f, g]$), if

$$\mathrm{Re}\,[Af, f]\leqq 0 \qquad \text{for all} \quad f\in\mathfrak{D}(A).$$

The following theorem is a slight modification of the Hille-Yosida theorem for contraction semigroups (comp. [**3**], theorem IX. 8).

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