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On characteristic classes of riemannian, conformal and projective foliations

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Introduction.

In this paper we shall study certain families of foliations with structures defined below. Our purpose is to prove a vanishing theorem for their characteristic classes.

Let M be a smooth n-manifold and TM its tangent bundle. Let E be an integrable smooth $(n-q)$-subbundle of TM. A foliated structure is then given on M by a system of local integrals $\mathcal{F}=\{f_\lambda\}$ of E, which satisfies the atlas condition: for each pair of local submersions $f_\lambda: U_\lambda \to \boldsymbol{R}^q$ and $f_\mu: U_\mu \to \boldsymbol{R}^q$, and for each $x \in U_\lambda \cap U_\mu$, there exists a local diffeomorphism $\gamma_{\mu\lambda}^x$ with $f_\mu = \gamma_{\mu\lambda}^x \circ f_\lambda$ in some neighborhood of x. $\mathcal{F}$ is called a G-foliation if we can take the $\{\gamma_{\mu\lambda}^x\}$ as local automorphisms of some G-structure. The principal object of this paper is a study of G-foliations associated with second order G-structures. Among those structures the conformal or projective ones have been known to be the most significant (cf. Ochiai [19]).

Our main theorem is stated as follows:

MAIN THEOREM. *Let $\mathcal{F}$ be a conformal (resp. projective) foliation of codimension q on a smooth manifold M (see § 1 for the precise definition). Suppose $q \geqq 3$ (resp. $q \geqq 2$). Then for the normal bundle $\nu = TM/E$ of $\mathcal{F}$, we have*

$$(*) \qquad \mathrm{Pont}^k(\nu ; \boldsymbol{R}) = 0 \quad \textit{for} \quad k > q,$$

where $\mathrm{Pont}^k(\nu ; \boldsymbol{R})$ contained in $H^k(M ; \boldsymbol{R})$ is the k-th homogeneous part of the Pontrjagin ring generated by the real Pontrjagin classes of ν.

Note that each riemannian foliation (see § 1) may be regarded as a conformal as well as a projective one. In the course of the proof of the Main Theorem, it can be seen that (*) holds for every riemannian foliation (cf. § 4). This is a theorem of Pasternack [21]. A riemannian foliation is a G-foliation associated with the riemannian structure, a first order G-structure, and is nothing but a foliation with bundle-like metric in the sense of Reinhart [23].

Our theorem may be illustrated as follows. As is well-known, smooth fibre bundles serve as trivial examples of foliations. It is not difficult to verify