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Some functors on Grothendieck exact sequences of type I

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Abstract. It has been shown that the exactness of Grothendieck exact sequences of type I, of abelian groups and their homomorphisms is preserved by the free functor F, the polynomial functors P_n, the n-th symmetric product functors Sp^n, and the direct limit functor $\varinjlim$. A necessary and sufficient condition has also been sought under which the polynomial functors Q_n preserve the exactness of Grothendieck exact sequences of type I.

1.1. Introduction.

Following M. N. Roby [6] we recall a sequence,

$$A \underset{\alpha_2}{\overset{\alpha_1}{\rightrightarrows}} B \xrightarrow{\beta} C \tag{1.2}$$

of sets A, B, C and functions α_1, α_2 and β on them a Grothendieck exact sequence of type I, if;

(a) β is onto,

(b) For any two elements b_1, b_2 of B the following two conditions are equivalent,

(i) $\beta(b_1)=\beta(b_2)$,

(ii) There exists an $a\in A$ such that: $\alpha_1(a)=b_1$, $\alpha_2(a)=b_2$.

In the language of M. Barr [1] a Grothendieck exact sequence of type I, may be called a right exact sequence. In the proof of the theorem (2.1) of next section, we shall use the well known definitions and facts given in [3].

For groups we modify the definitions of Grothendieck exact sequences so as to that functions are replaced by group homomorphisms and consequently onto functions correspond to epimorphisms.

In this paper we have studied the behavior of such sequences with abelian groups under the free functor F, the polynomial functors P_n, Q_n, the n-th symmetric product functors Sp^n [2, 5] and the direct limit functor $\varinjlim$. It has